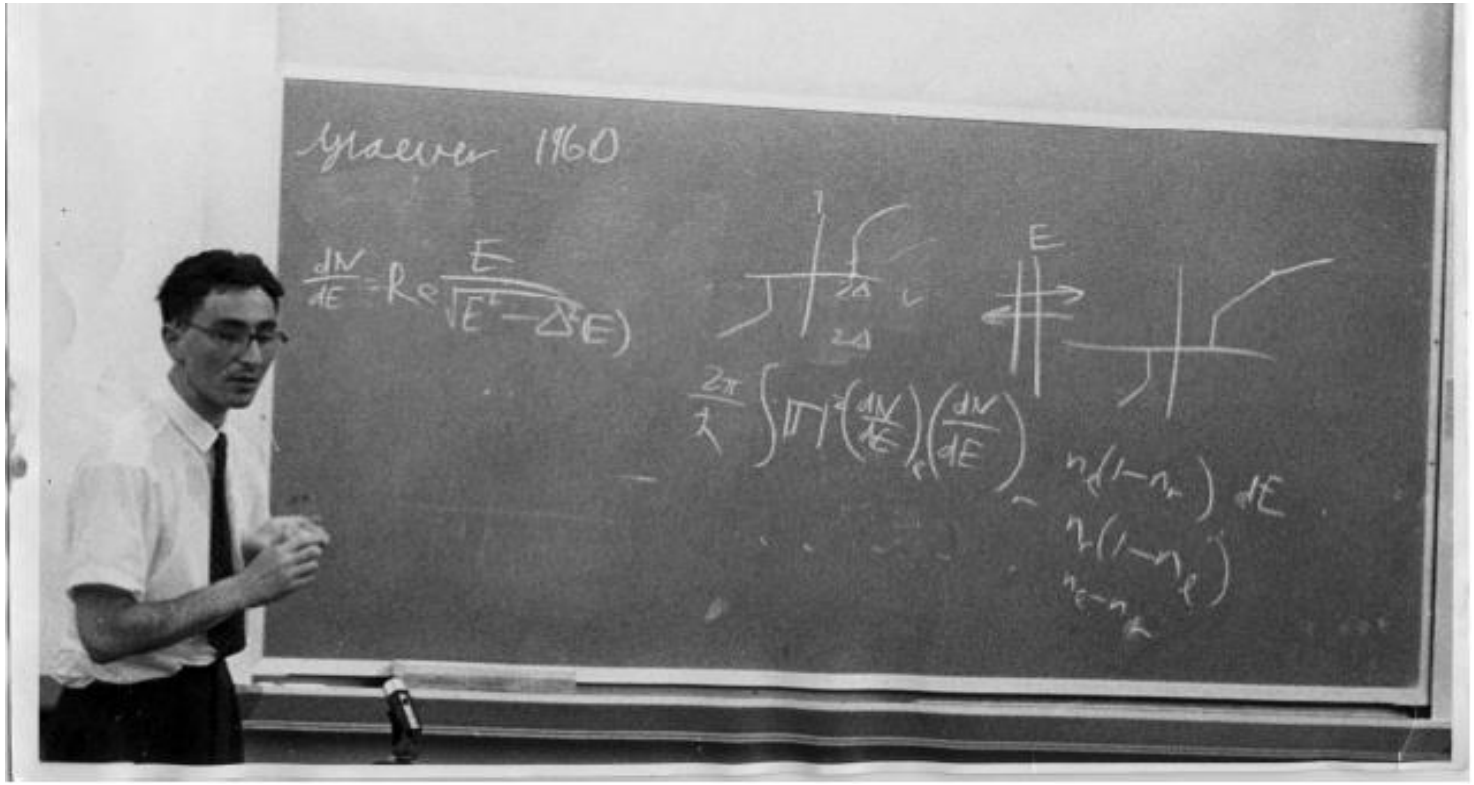
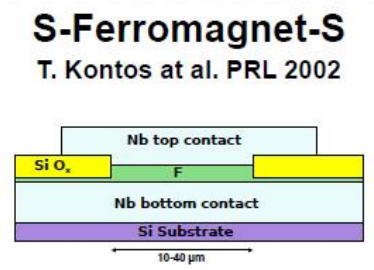
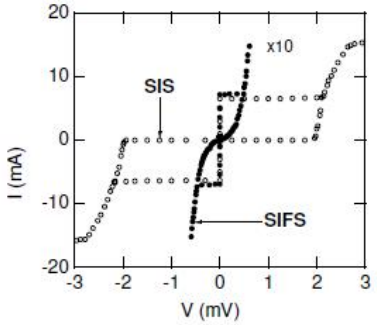
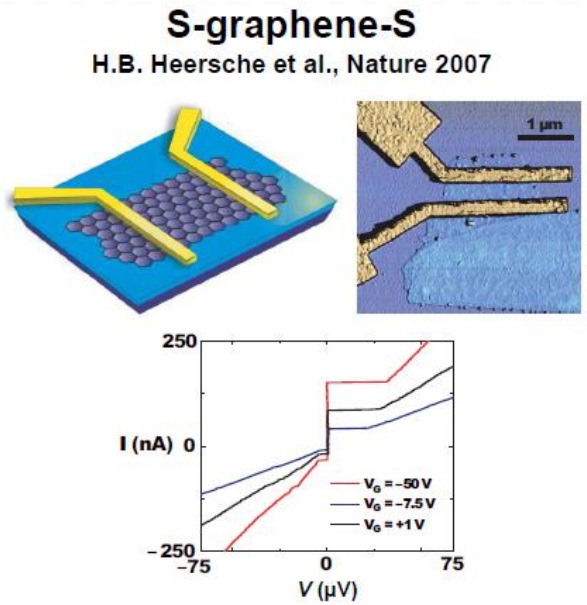
LECTURES ON MESOSCOPIC SUPERCONDUCTIVITY

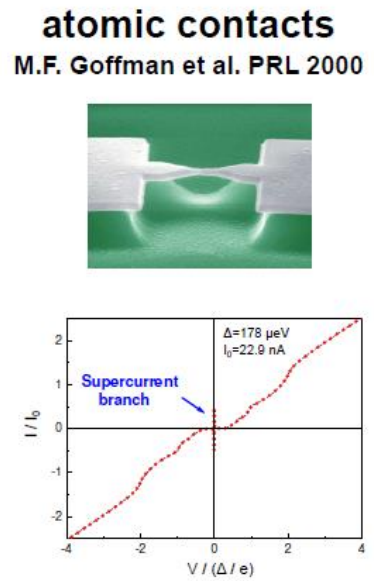


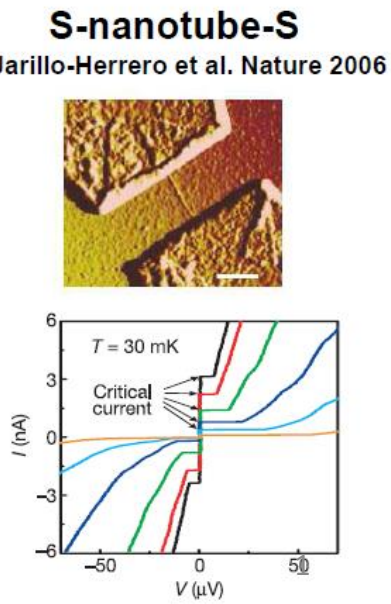
Brian Josephson explains tunneling between superconductors (circa 1962)











LEANDRO TOSI

QUANTRONICS GROUP

SCHOOL ON QUANTUM MATERIAL AND WORKSHOP ON VORTEX BEHAVIOR IN UNCONVETIONAL SUPERCONDUCTORS

BRAGA, OCT 2018

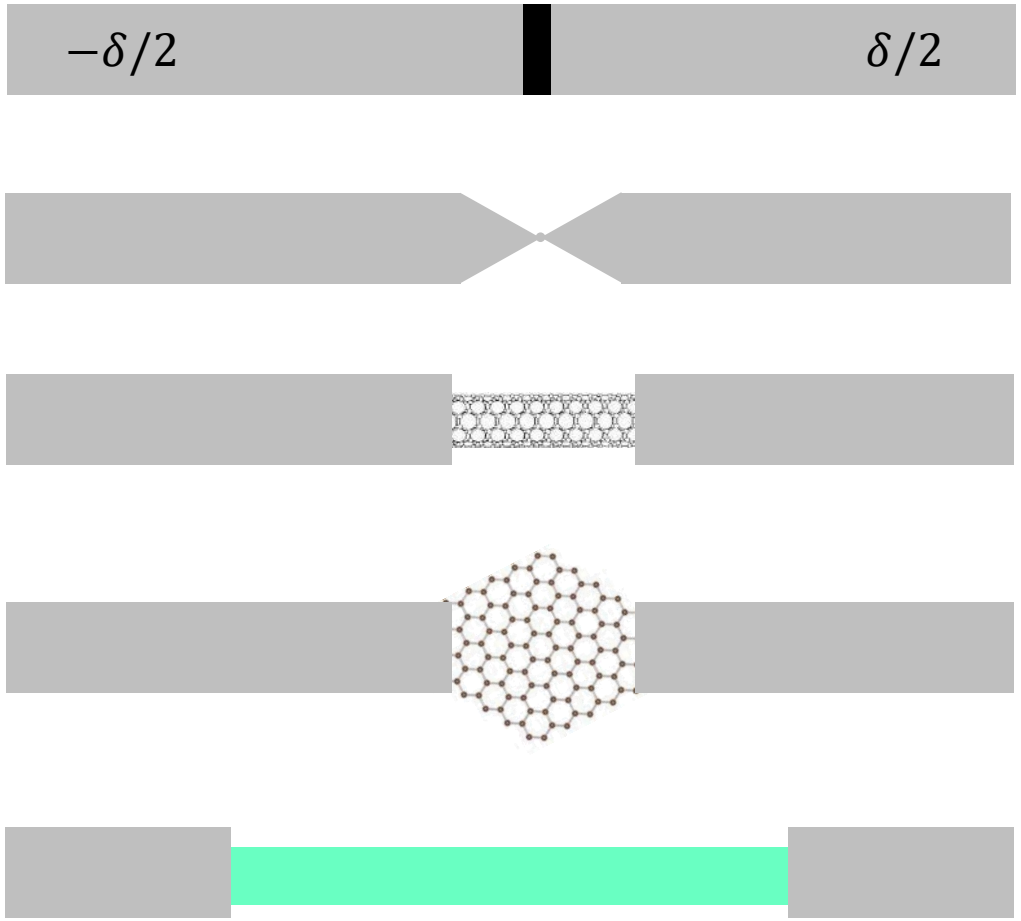
THE JOSEPHSON EFFECT

$|\Delta|e^{i\varphi}$

THE JOSEPHSON EFFECT

$$I = I_0 \sin\delta$$

THE JOSEPHSON EFFECT…
IN ALL TYPES OF WEAK LINKS



$$I = I_0 \sin\delta$$

$$I = f_{2\pi}(\delta)$$

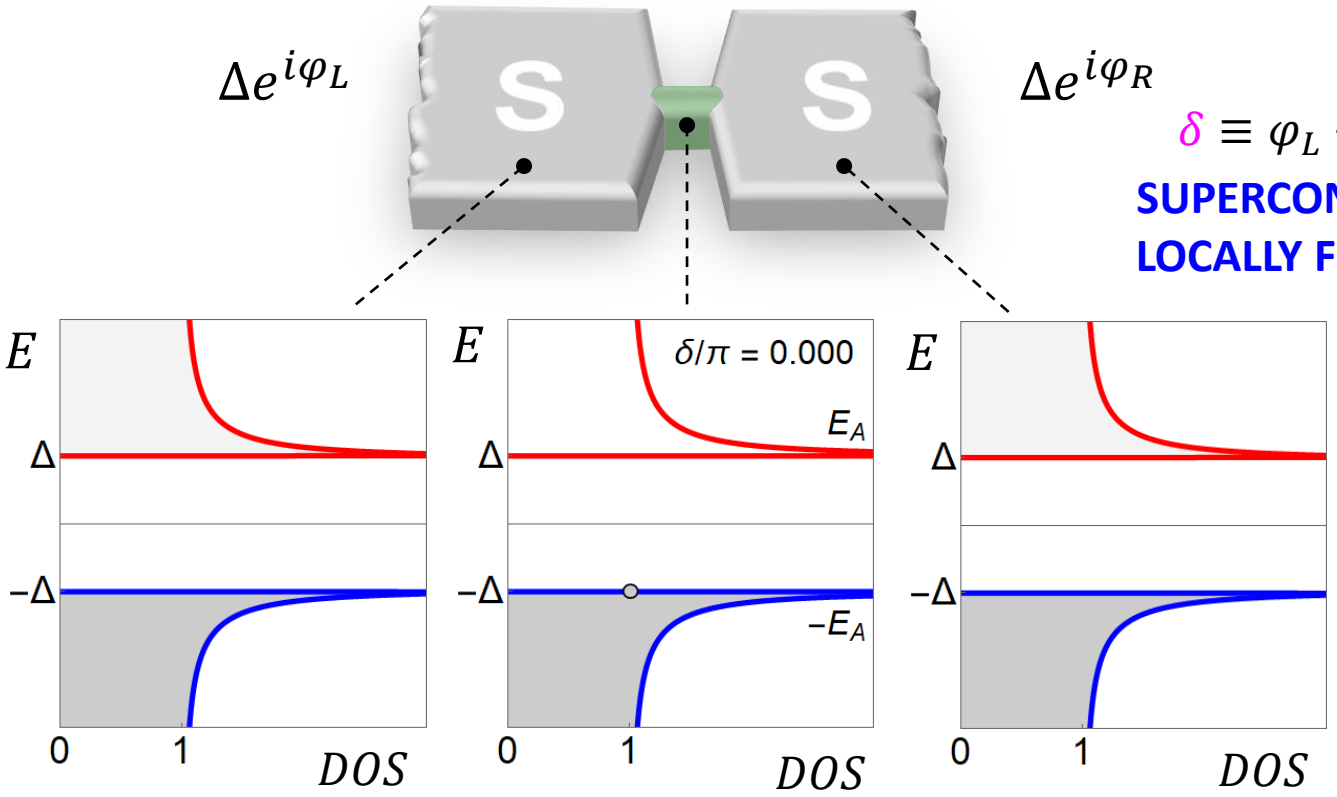
mesoscopic description:

➔ **Andreev bound states**

SIMPLE OUTLINE

1. ORIGIN OF ABS
2. PROBE ABS
3. USE ABS AS A QUBIT

ELEMENTARY SC WEAK-LINK

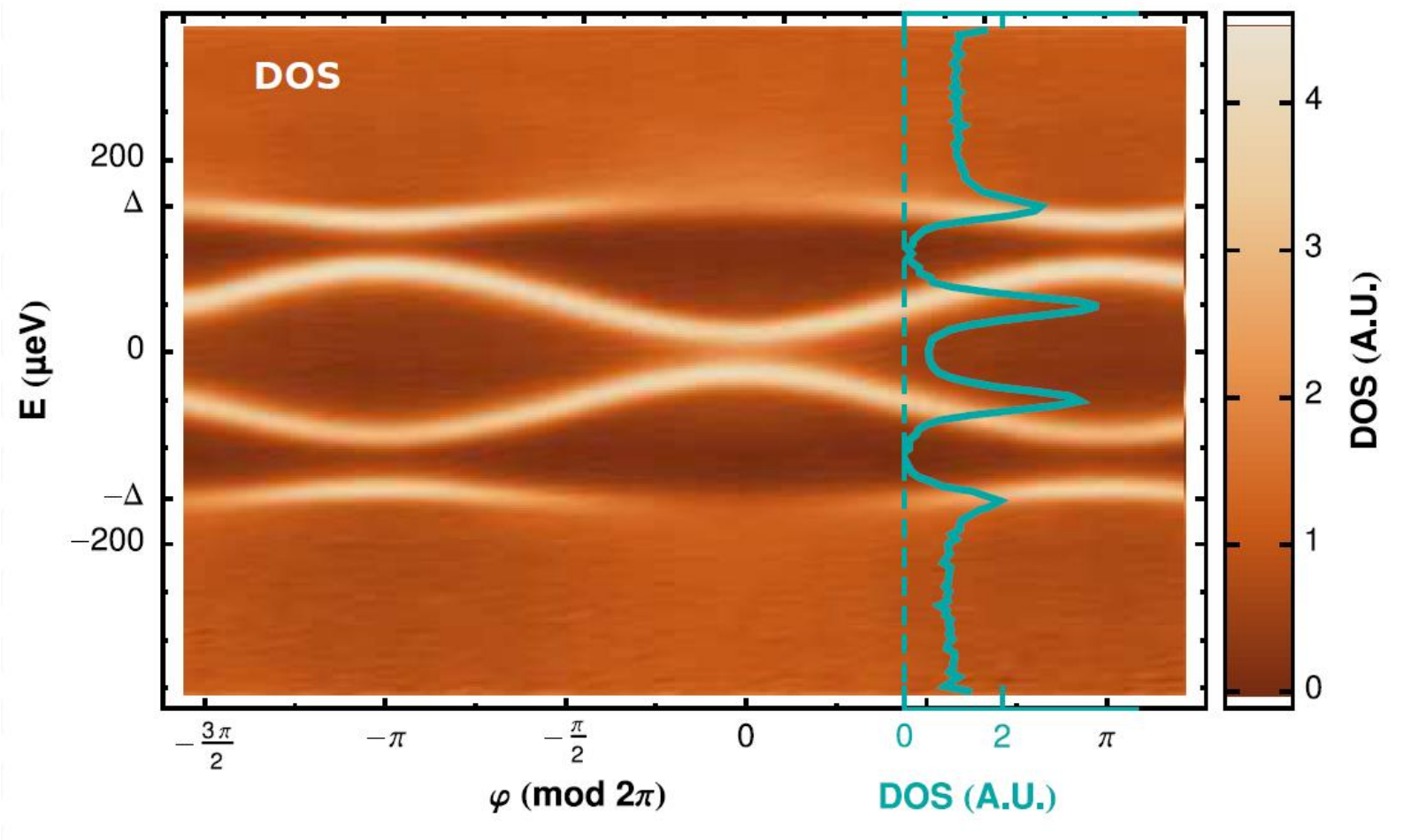


$\delta \equiv \varphi_L - \varphi_R \neq 0$

SUPERCONDUCTIVITY LOCALLY FRUSTRATED

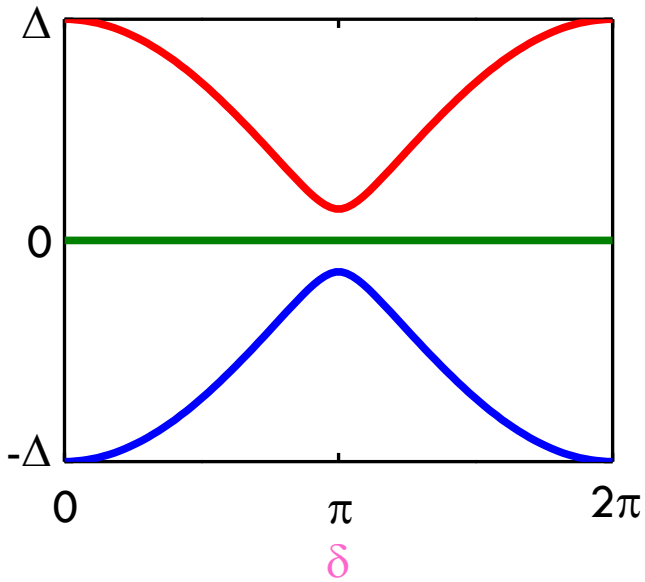
SIMPLE OUTLINE

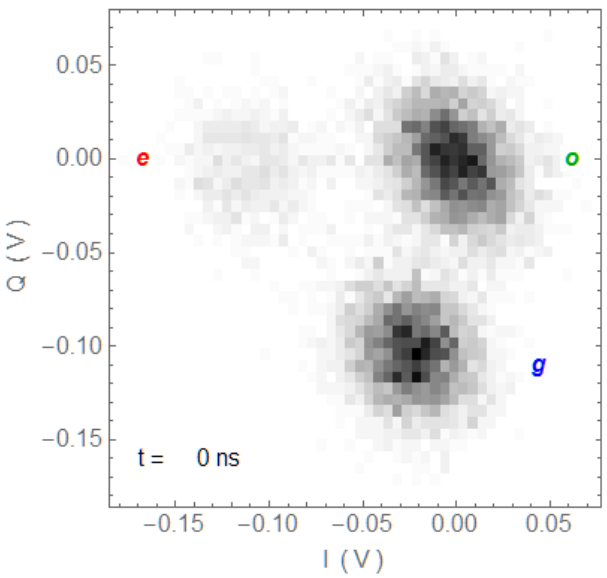
1. ORIGIN OF ABS
2. PROBE ABS
3. USE ABS AS A QUBIT

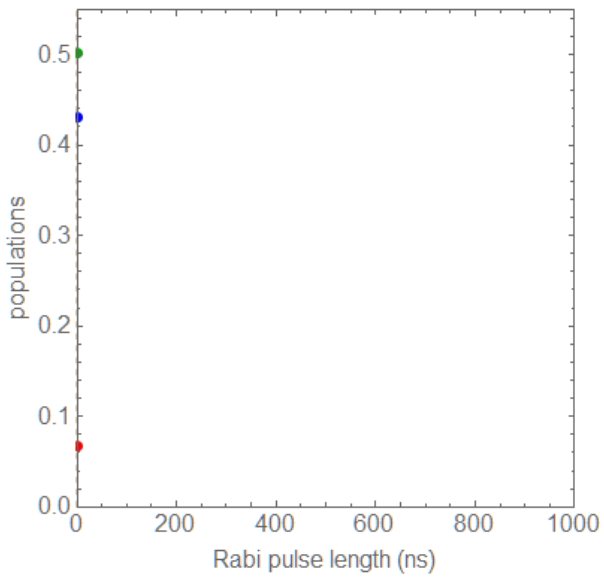


SIMPLE OUTLINE

1. ORIGIN OF ABS
2. PROBE ABS
3. USE ABS AS A QUBIT







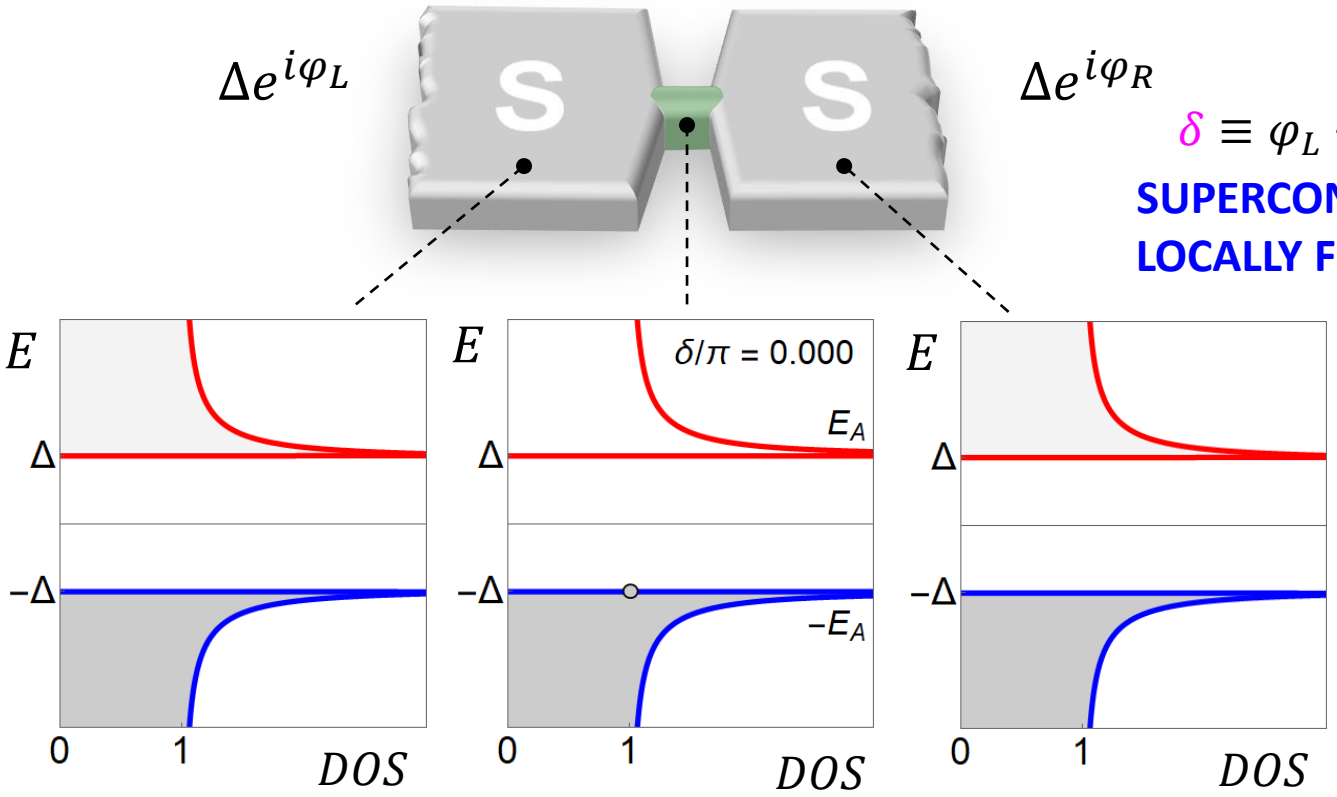
SIMPLE OUTLINE

1. ORIGIN OF ABS

2. PROBE ABS

3. USE ABS AS A QUBIT

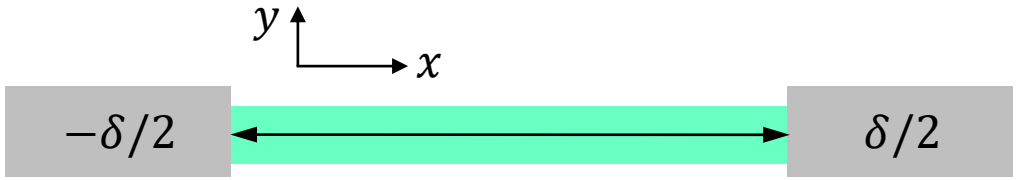
ELEMENTARY SC WEAK-LINK



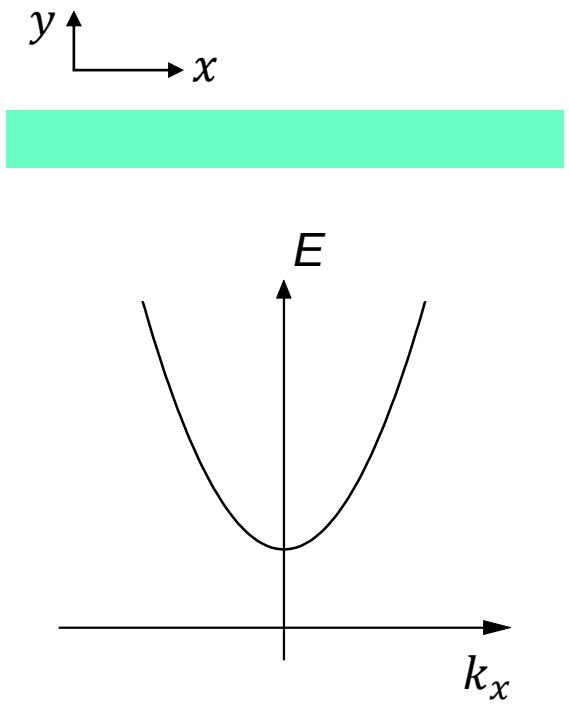
$\delta \equiv \varphi_L - \varphi_R \neq 0$

SUPERCONDUCTIVITY LOCALLY FRUSTRATED

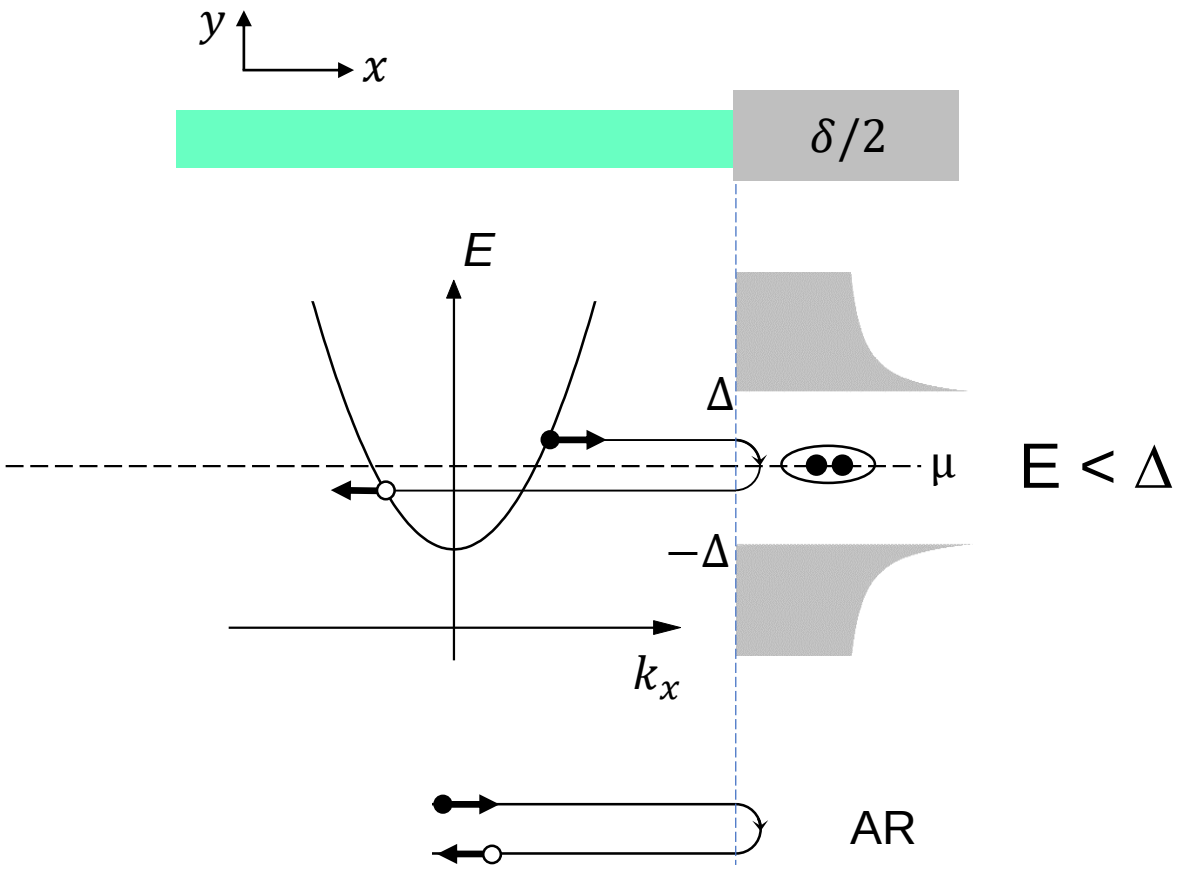
A SCATTERING PROBLEM



A SINGLE BAND

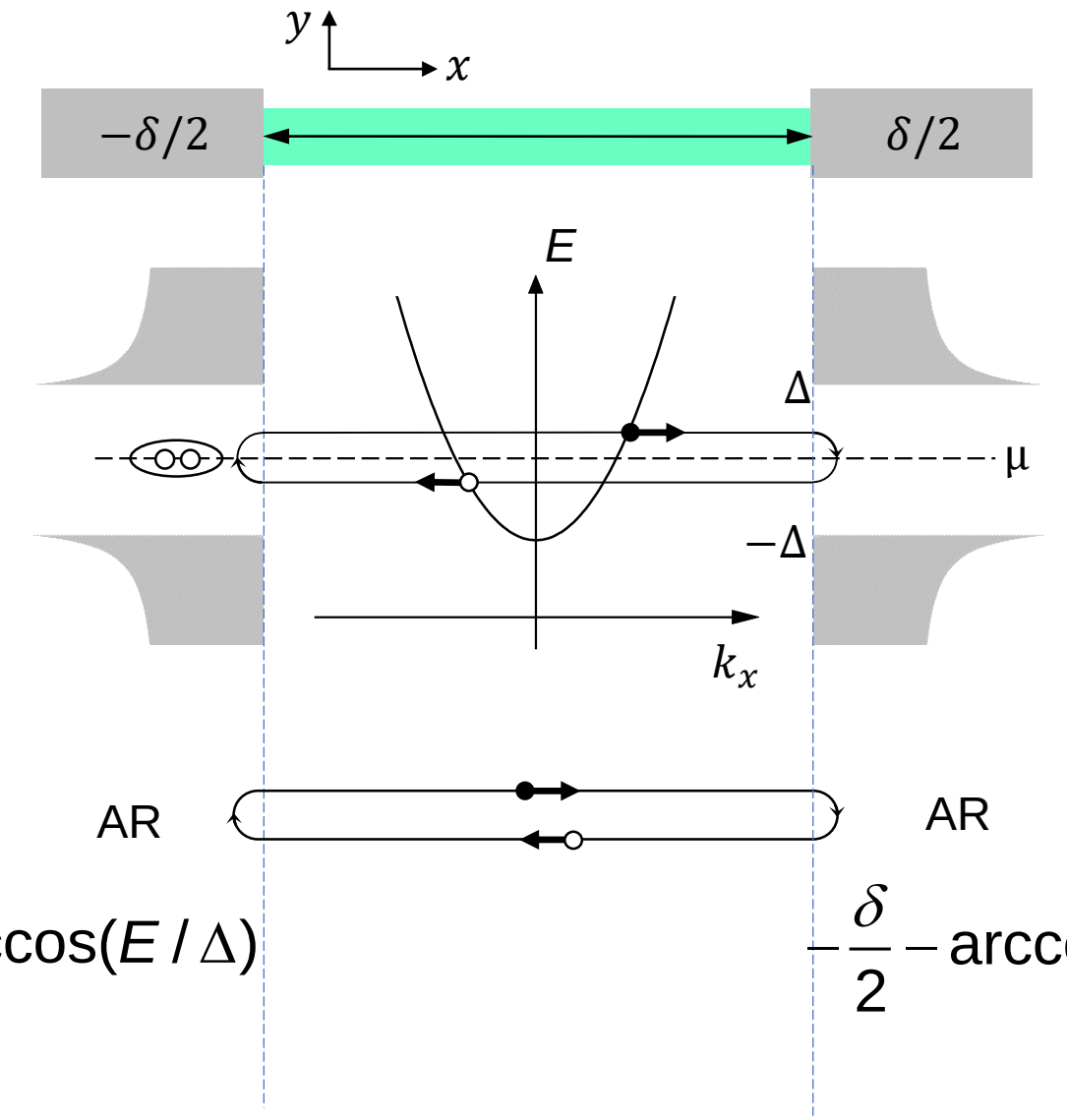


ANDREEV REFLECTION



$$\text{phase } -\frac{\delta}{2} - \arccos(E/\Delta)$$

BALLISTIC CHANNEL

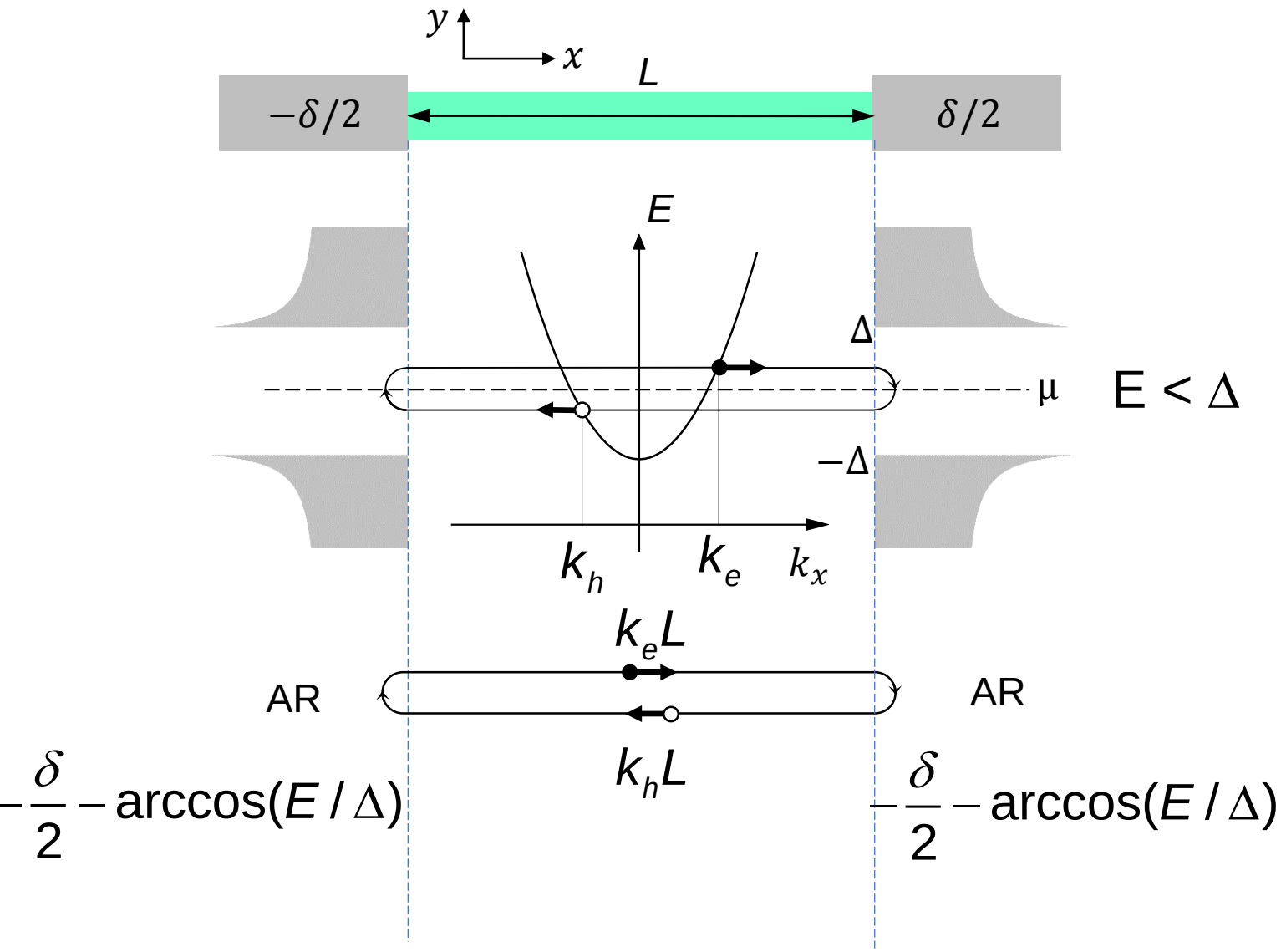


$E < \Delta$

$$-\frac{\delta}{2} - \arccos(E/\Delta)$$

$$-\frac{\delta}{2} - \arccos(E/\Delta)$$

BALLISTIC CHANNEL



$E < \Delta$

$$k_e = k_F + \frac{E}{\hbar V_F}$$

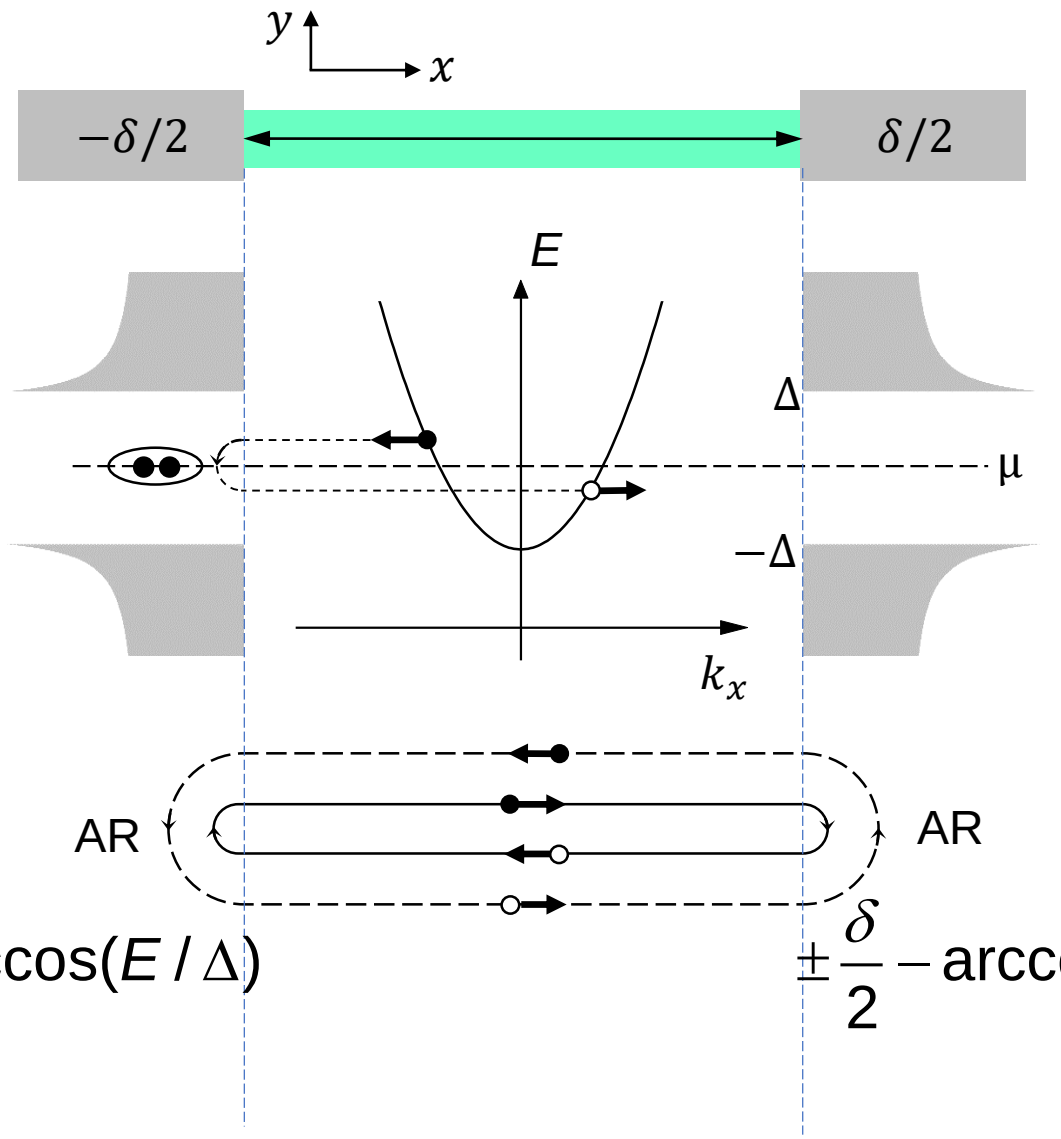
$$k_h = -k_F + \frac{E}{\hbar V_F}$$

$$k_eL + k_hL = 2\lambda\varepsilon$$

$$\lambda = \frac{L}{\hbar V_F / \Delta}; \quad \varepsilon = E/\Delta$$

$$\boxed{-\delta - 2\arccos\varepsilon + 2\lambda\varepsilon = 0 \ (\mathrm{mod}\, 2\pi)}$$

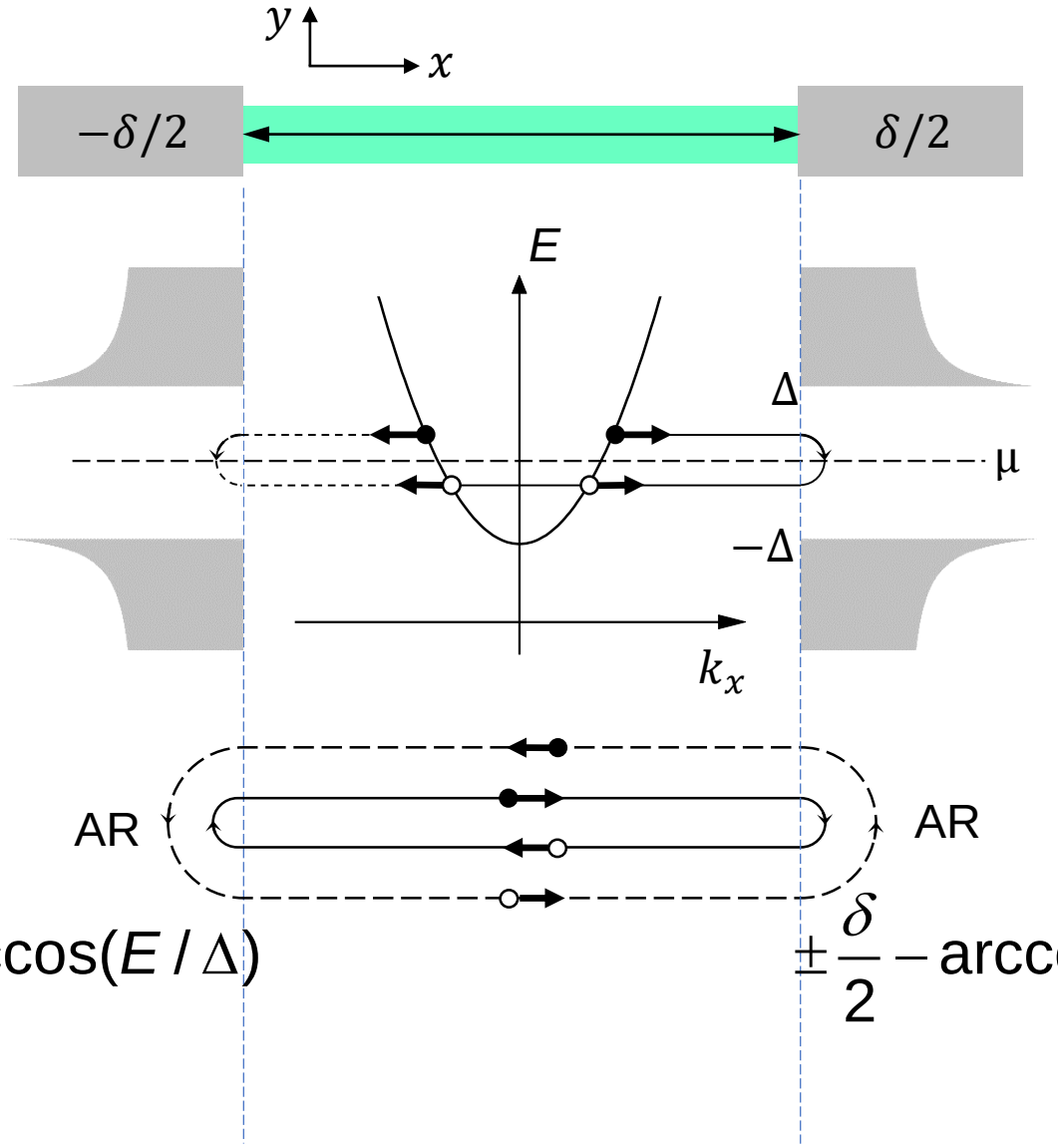
BALLISTIC CHANNEL



$$\pm\frac{\delta}{2}-\arccos(E/\Delta)$$

$$\pm\frac{\delta}{2}-\arccos(E/\Delta)$$

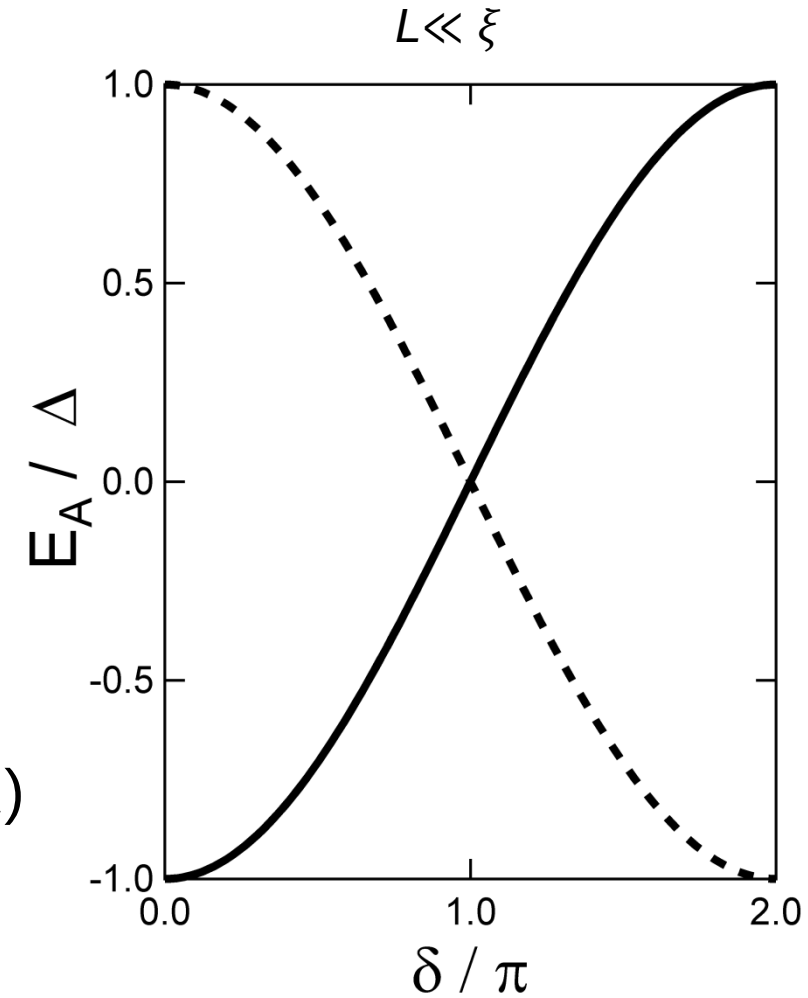
ABS FOR A SINGLE BALLISTIC CHANNEL



$$\pm\frac{\delta}{2} - \arccos(E/\Delta)$$

$$\pm\frac{\delta}{2} - \arccos(E/\Delta)$$

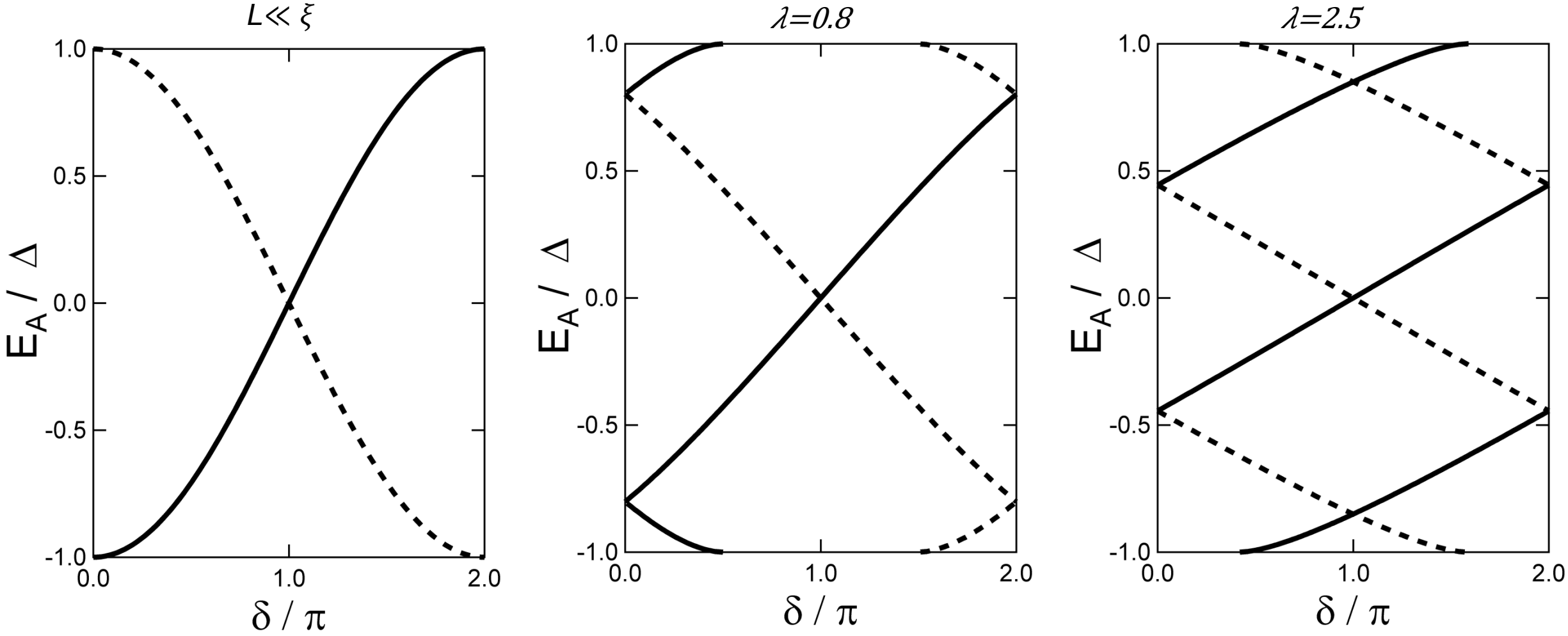
$$\pm\delta - 2\arccos\varepsilon + 2\lambda\varepsilon = 0 \pmod{2\pi}$$



EVOLUTION OF ABS FROM SHORT TO LONG JUNCTION, SINGLE BALLISTIC CHANNEL

$$\pm\delta - 2\arccos(E/\Delta) = 0 \pmod{2\pi}$$

$$\pm\delta - 2\arccos\varepsilon + 2\lambda\varepsilon = 0 \pmod{2\pi}$$

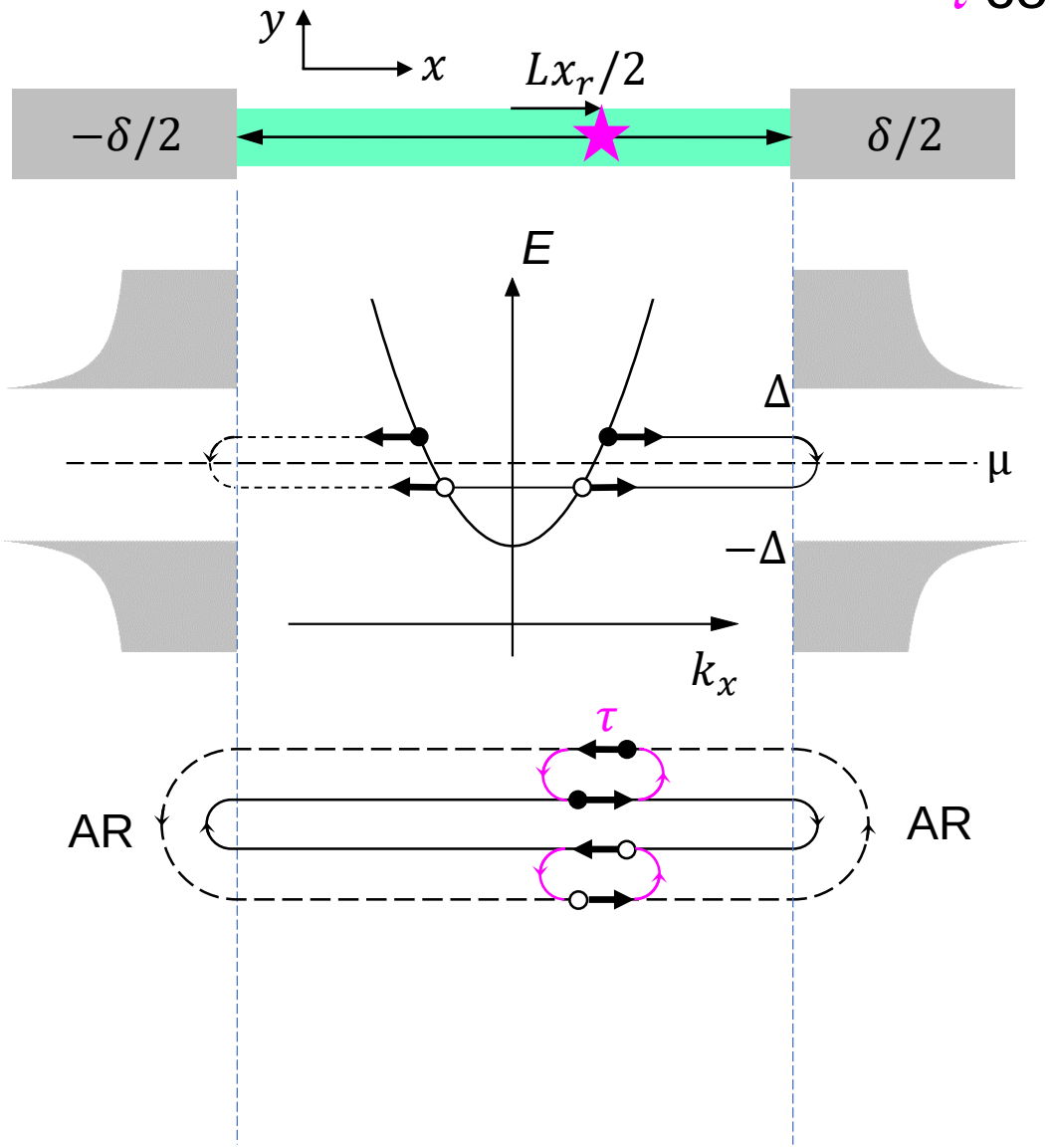


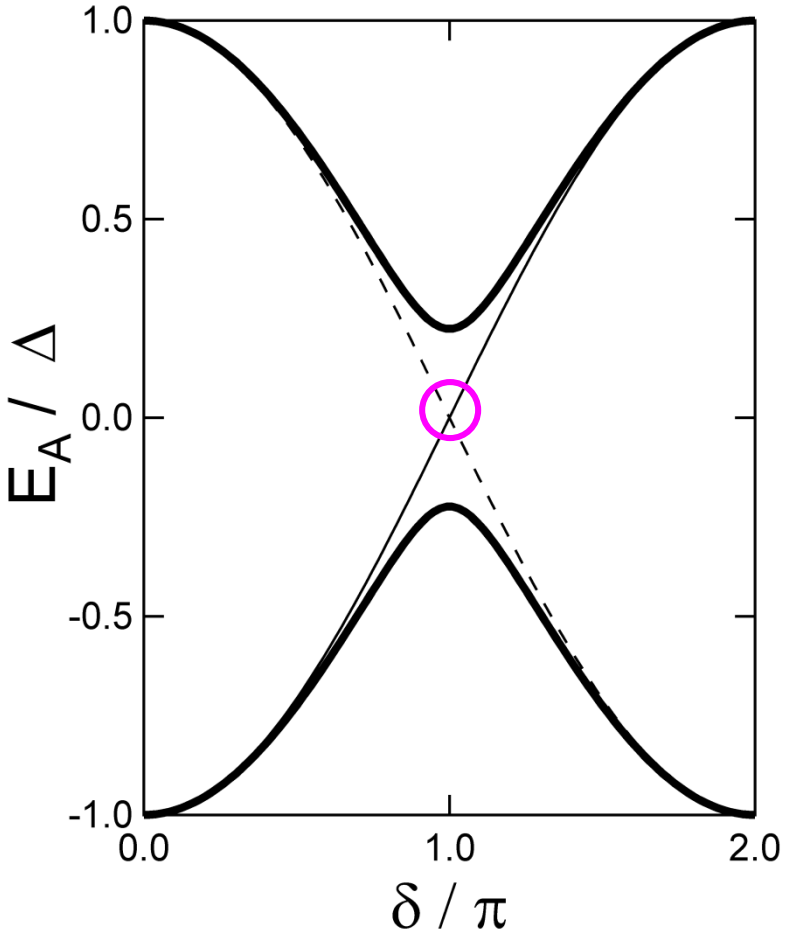
ABS FOR A SINGLE CHANNEL WITH SCATTERER

$$\tau\cos(\delta)+(1-\tau)\cos(2\lambda\varepsilon x_r)=\cos(2\arccos\varepsilon-2\lambda\varepsilon)$$

$$L \ll \xi$$

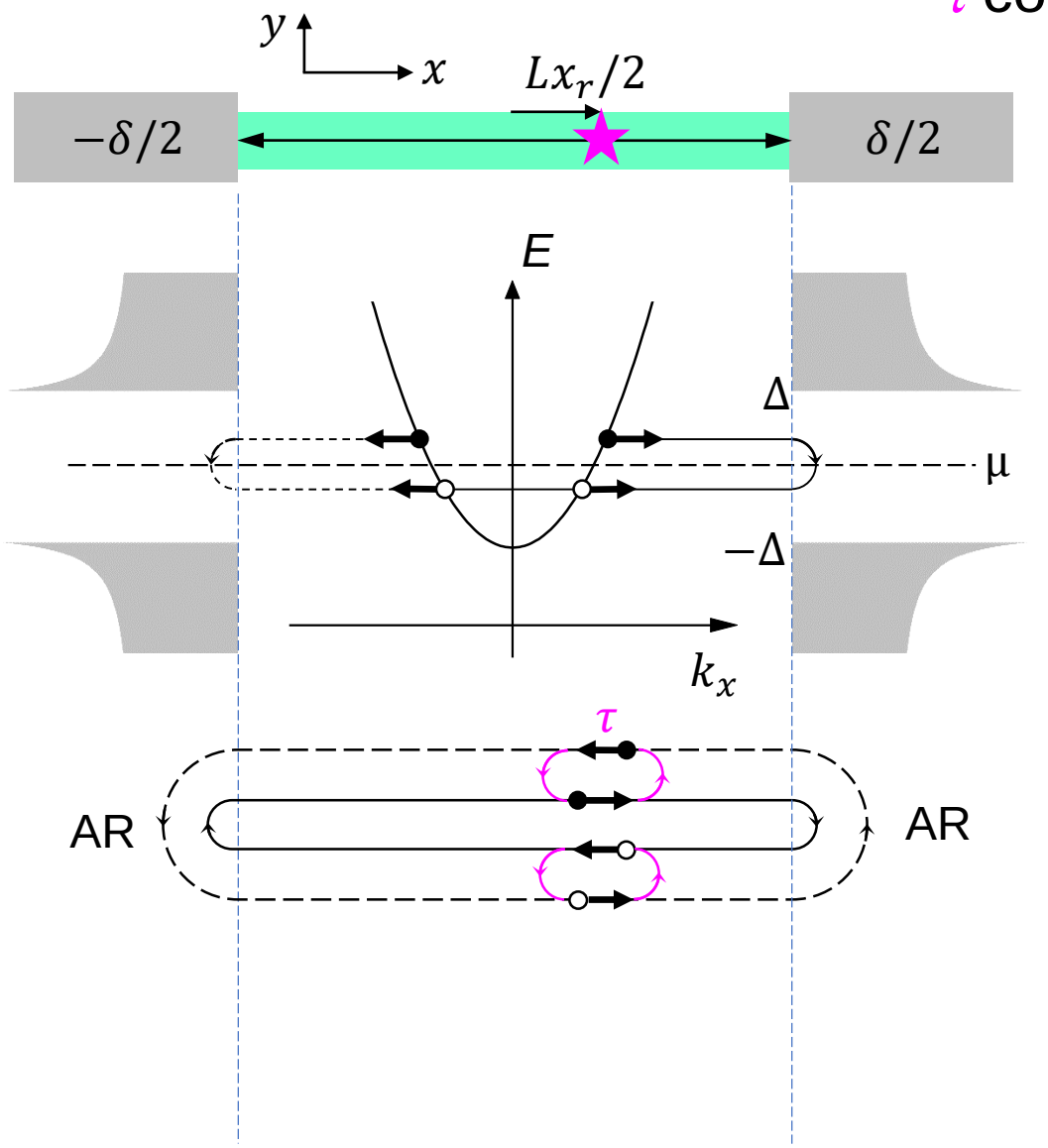
$$E=\Delta\sqrt{1-\tau\sin^2(\delta/2)}$$

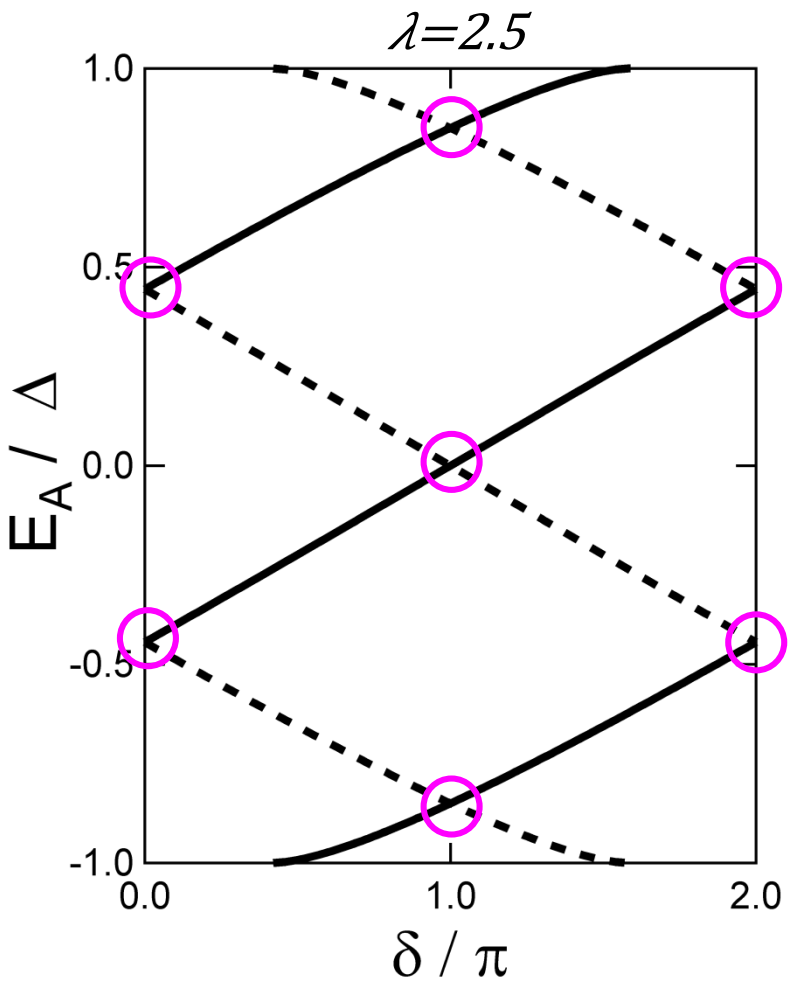




ABS FOR A SINGLE CHANNEL WITH SCATTERER

$$\tau\cos(\delta)+(1-\tau)\cos(2\lambda\varepsilon x_r)=\cos(2\arccos\varepsilon-2\lambda\varepsilon)$$

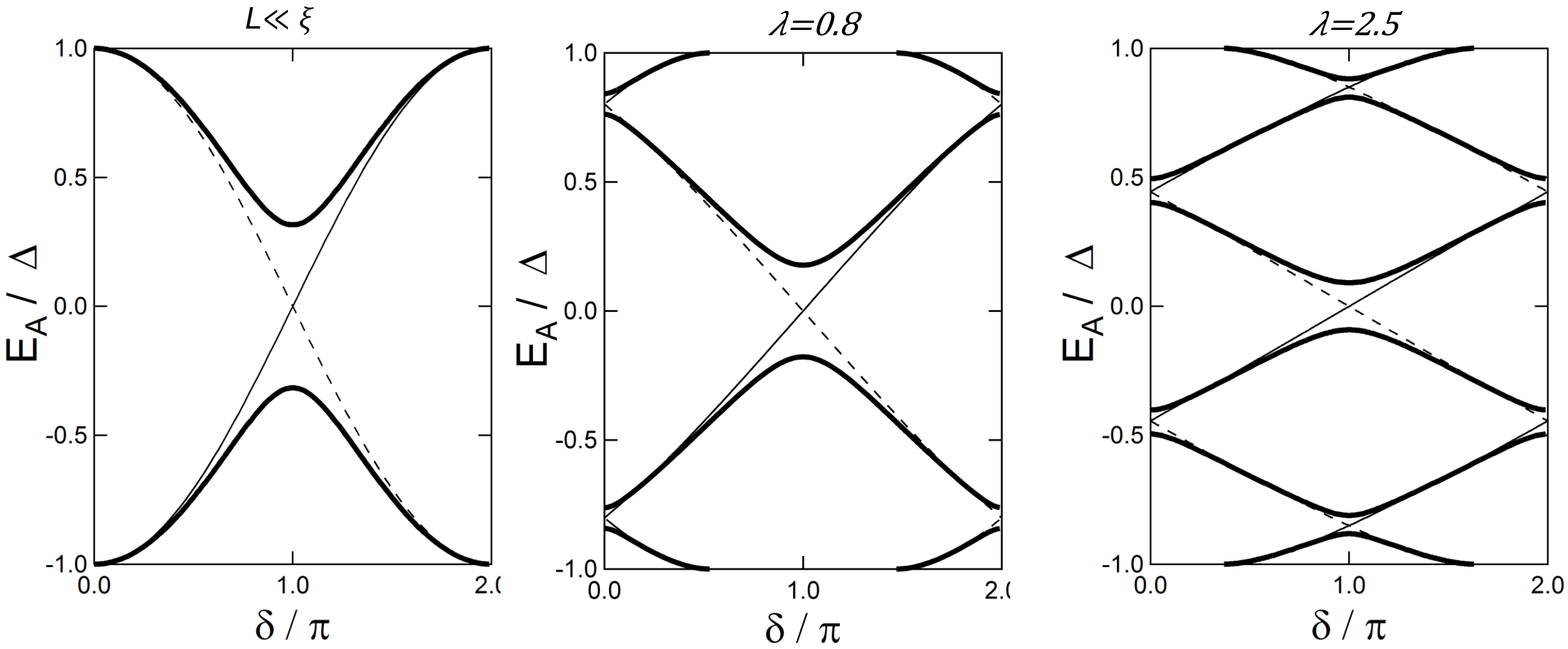




EVOLUTION OF ABS FROM SHORT TO LONG JUNCTION, SINGLE CHANNEL WITH SCATTERER

$\tau = 0.9$

$$\tau\cos(\delta) + (1-\tau)\cos(2\lambda\varepsilon x_r) = \cos(2\arccos\varepsilon - 2\lambda\varepsilon)$$

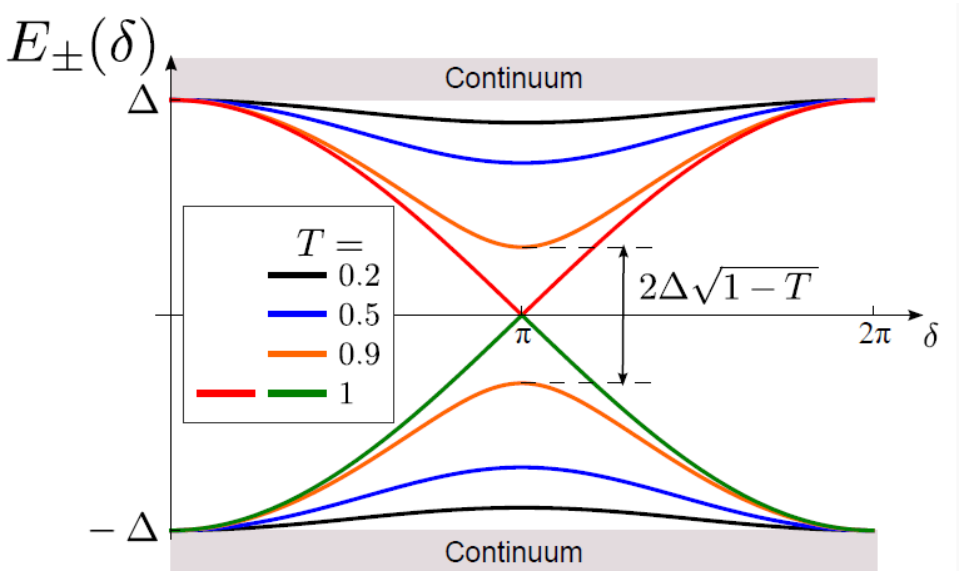


ABS IN A SHORT CHANNEL

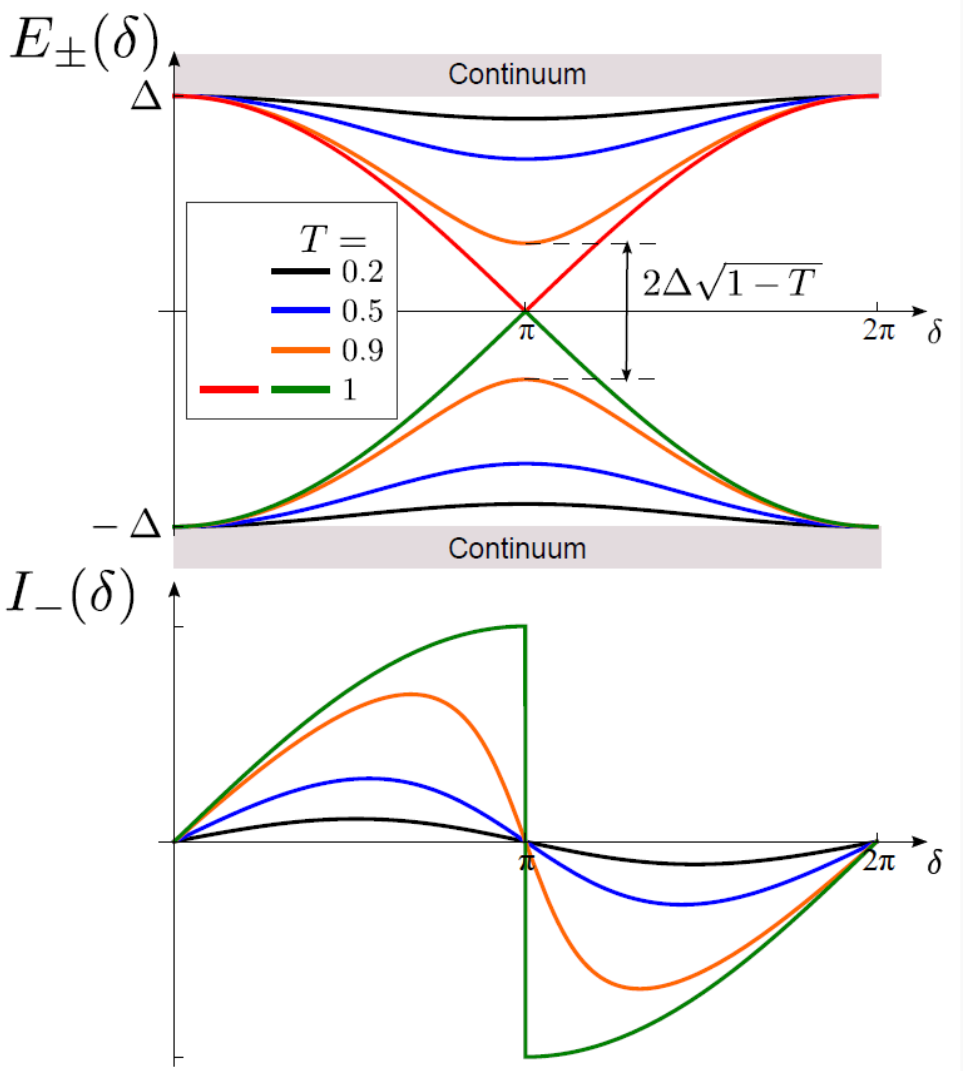
$L \ll \xi$

$$\tau \cos(\delta) + (1-\tau) = \cos(2 \arccos \varepsilon)$$

$$E = \Delta\sqrt{1-\tau \sin^2(\delta/2)}$$



SUPERCURRENT OF ABS IN A SHORT CHANNEL



ENERGY

$$E_\pm(\delta,\tau) = \pm\Delta\sqrt{1-\tau\sin^2(\delta/2)}$$

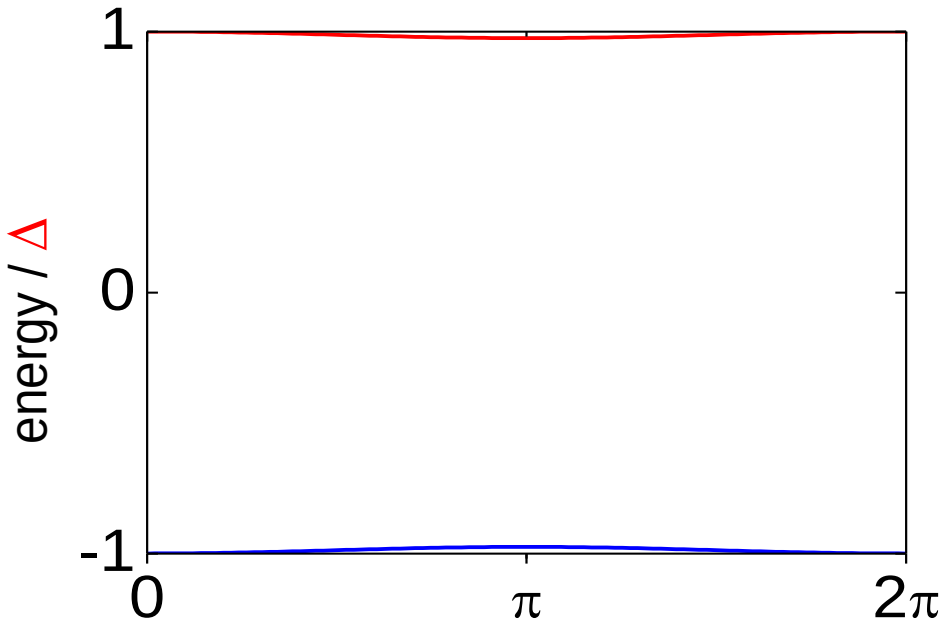
SUPERCURRENT

$$I_\pm(\delta,\tau) = \frac{2\pi}{\phi_0}\frac{\partial E}{\partial\delta}$$

$$= \pm\frac{e\Delta}{2\hbar}\tau\frac{\sin\delta}{\sqrt{1-\tau\sin^2(\delta/2)}}$$

STANDARD JOSEPHSON EFFECT IN ABS PICTURE

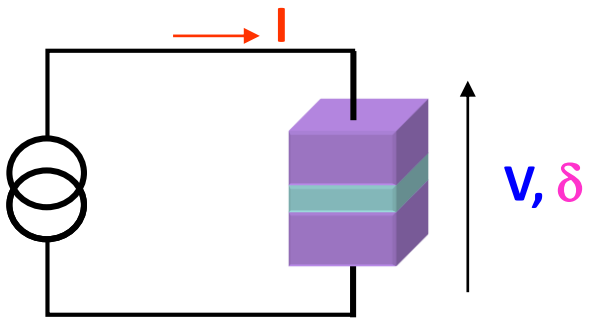
Many channels with $\tau \ll 1$



NO NEED TO CONSIDER EXCITED STATE IF

$$kT, \hbar\omega \ll 2\Delta$$

JOSEPHSON ENERGY FOR TUNNEL JUNCTION



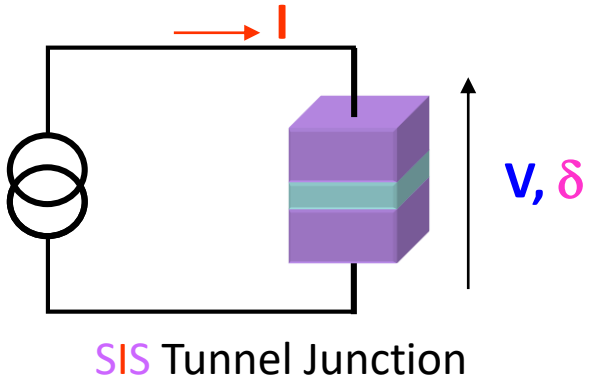
a very large number $N \sim \dfrac{A}{\lambda_F^2}$ of weakly transmitted $\tau \leq 10^{-6}$ channels

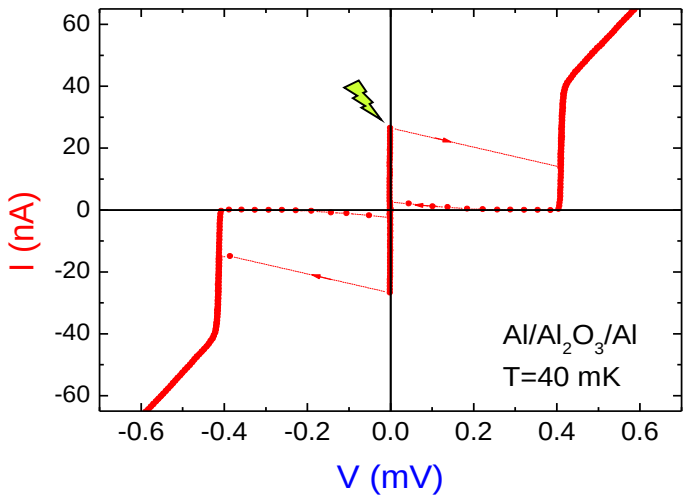
JOSEPHSON ENERGY in ground state

$$E_g(\delta) = \lim_{\tau_n \to 0} \left\{ -\Delta \sum_n \sqrt{1 - \tau_n \sin^2(\delta/2)} \right\} \approx -\Delta \sum_n \left[\frac{\tau_n}{4} \cos\delta + \frac{\tau_n}{4} - 1 \right]$$

$$\boxed{E_g(\delta) \approx -E_J \cos\delta \quad \text{with} \quad E_J = \frac{\Delta}{4} \frac{G_N}{G_0} = \frac{\Delta}{4} \frac{R_K}{R_N}}$$

JOSEPHSON CURRENT FOR TUNNEL JUNCTION





JOSEPHSON RELATION

$$I_{\pm}(\delta,\tau) = \pm\sum_n \frac{e\Delta}{2\hbar}\tau_n \sin\delta = I_0 \sin\delta$$

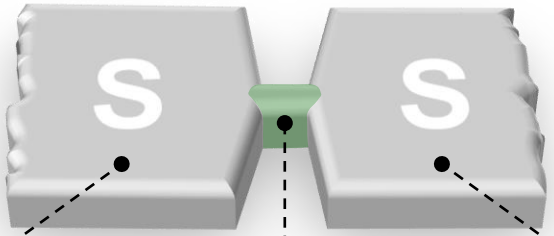
AMBEGAOKAR-BARATOFF

$$I_0 = \frac{e\Delta}{2\hbar}\sum_n \tau_n = \frac{\pi}{4}\frac{2\Delta}{e}G_N$$

ELEMENTARY SC WEAK-LINK

SINGLE CHANNEL WEAK LINK OF TRANSMISSION τ

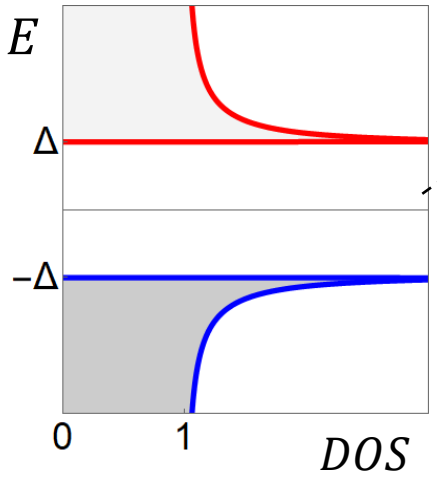
$\Delta e^{i\varphi_L}$

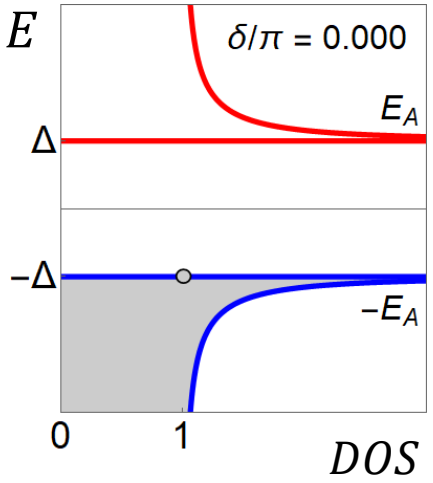


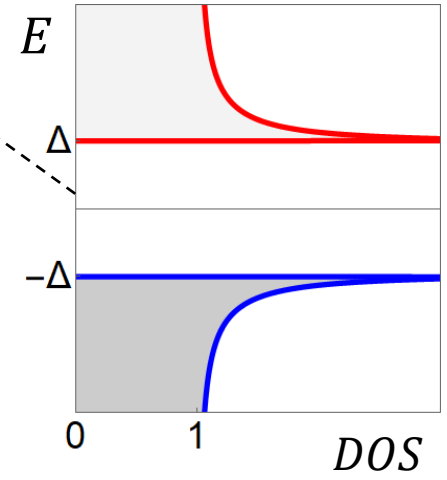
$\Delta e^{i\varphi_R}$

$\delta \equiv \varphi_L - \varphi_R \neq 0$

SUPERCONDUCTIVITY LOCALLY FRUSTRATED



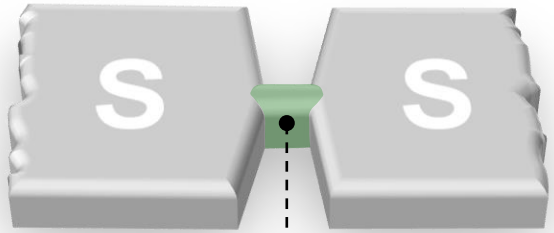




ELEMENTARY SC WEAK-LINK

SINGLE CHANNEL WEAK LINK OF TRANSMISSION τ

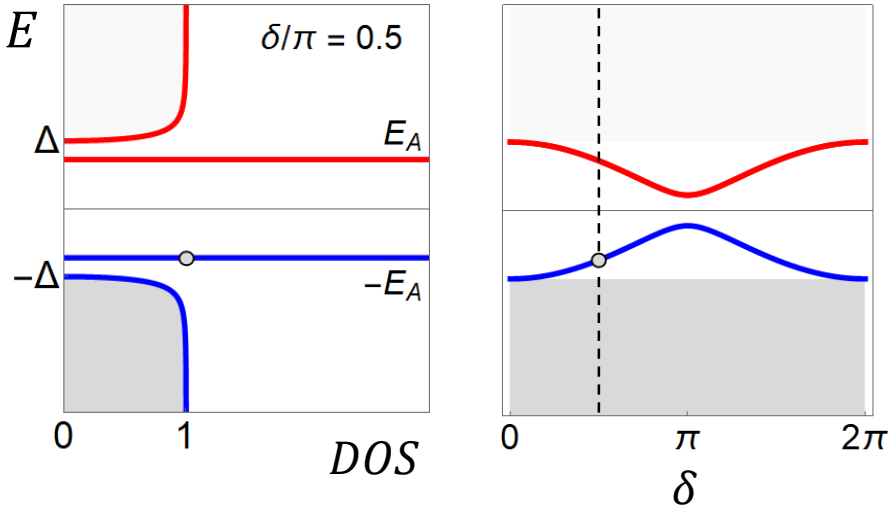
$\Delta e^{i\varphi_L}$



$\Delta e^{i\varphi_R}$

$$\delta \equiv \varphi_L - \varphi_R \neq 0$$

SUPERCONDUCTIVITY LOCALLY FRUSTRATED



ANDREEV BOUND STATES

A SPIN DEGENERATE LOCALIZED QUASIPARTICLE STATE

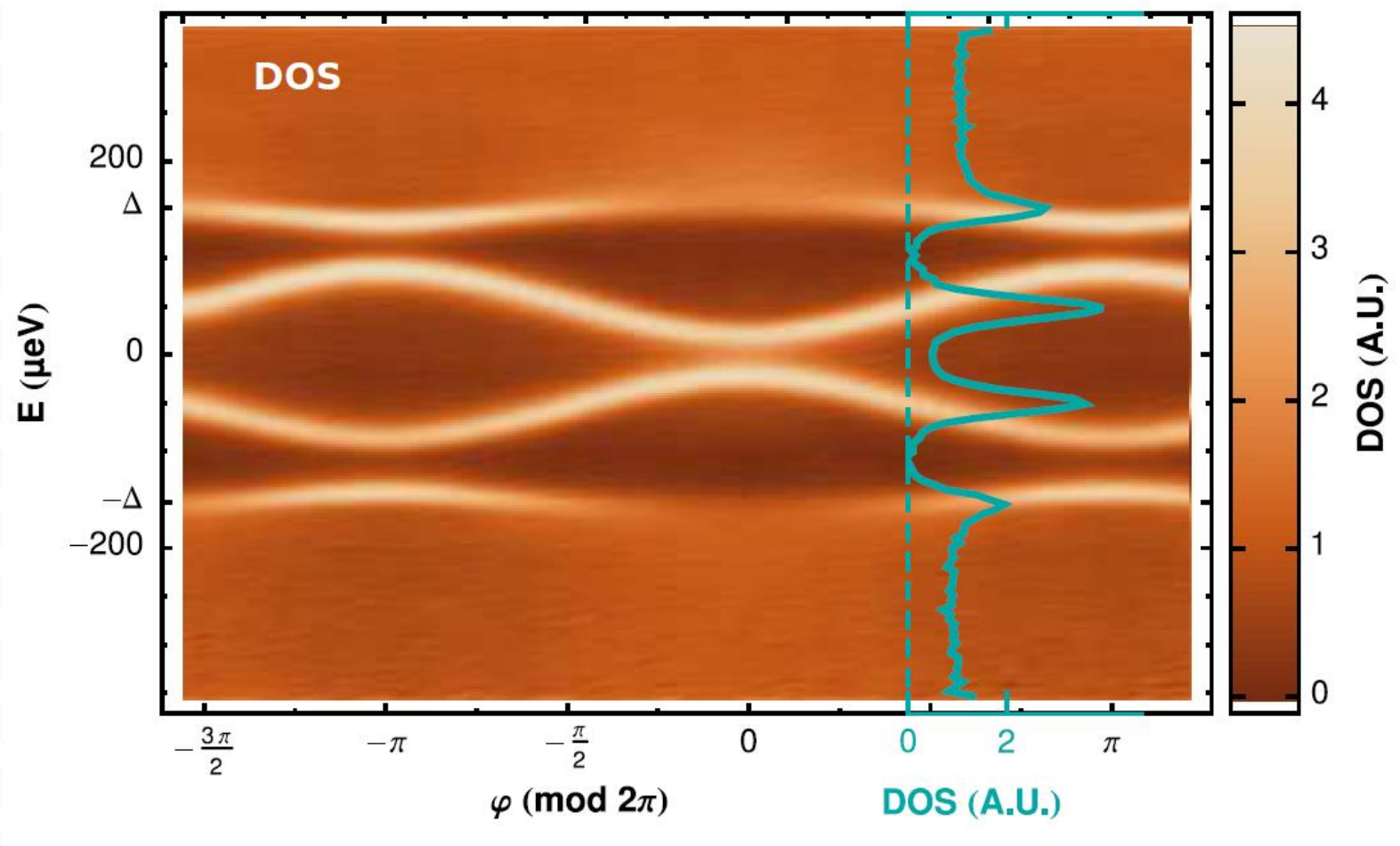
$$E_A = \Delta\sqrt{1-\tau\sin^2(\delta/2)}$$

SIMPLE OUTLINE

1. ORIGIN OF ABS

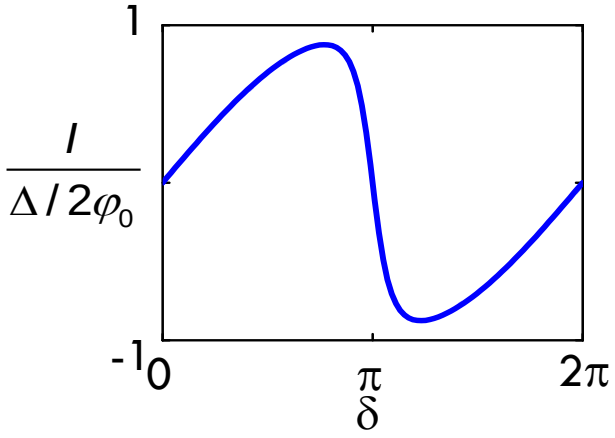
2. PROBE ABS

3. USE ABS AS A QUBIT



PROBE MESOSCOPIC THEORY OF JOSEPHSON EFFECT

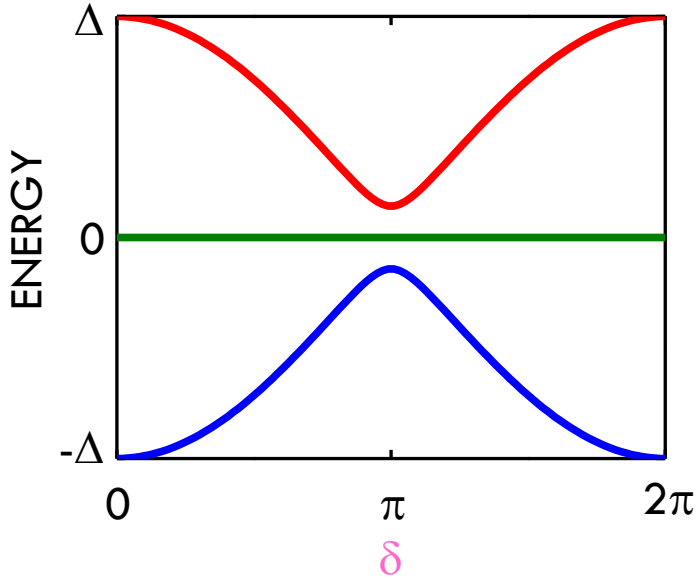
- probe $I(\delta)$ in $|g\rangle$



- Spectroscopy of ABS
- Drive coherent oscillations between states

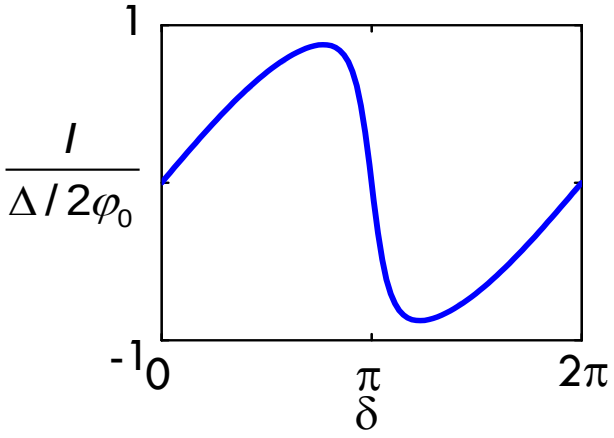
➔ requires:

1. Well-controlled system:
   few channels, measurable transmissions
2. Phase biasing
3. Detection of transitions



PROBE MESOSCOPIC THEORY OF JOSEPHSON EFFECT

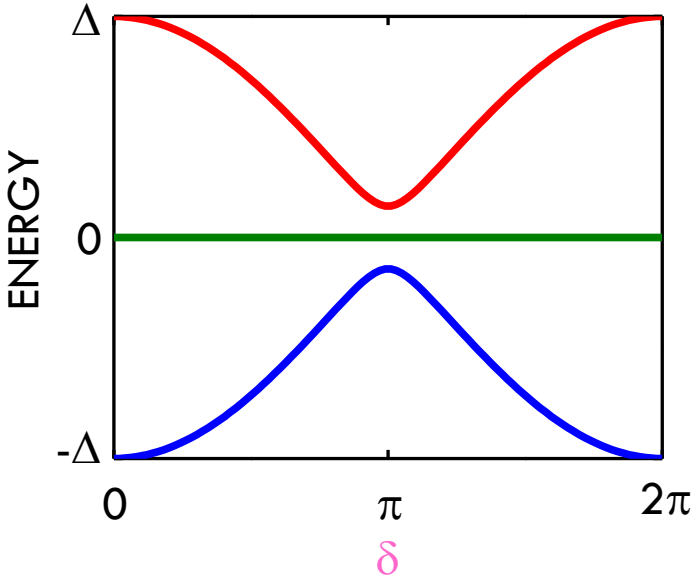
- probe $I(\delta)$ in $|g\rangle$



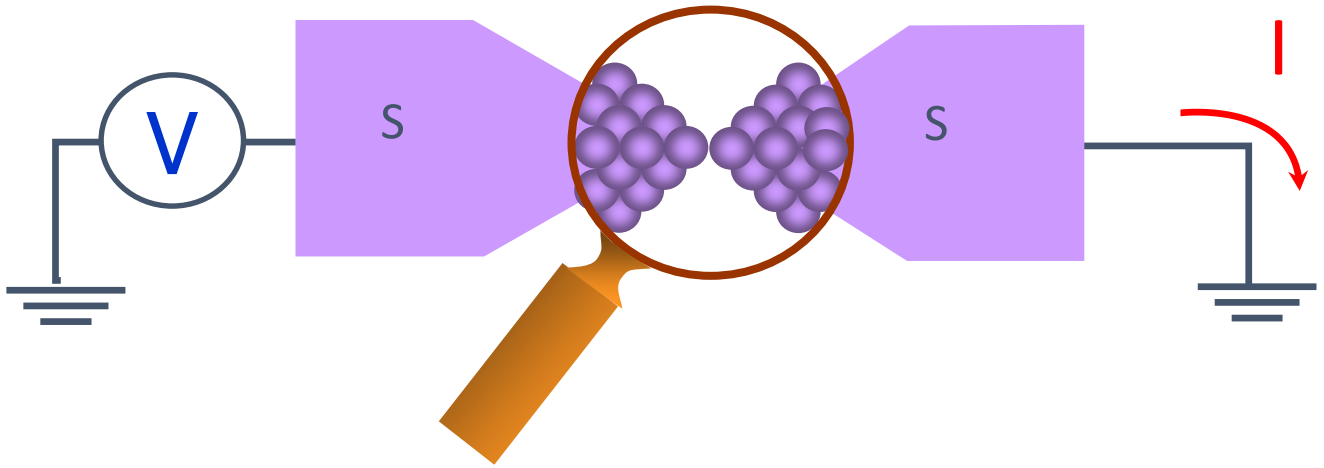
- Spectroscopy of ABS
- Drive coherent oscillations between states

➔ requires:

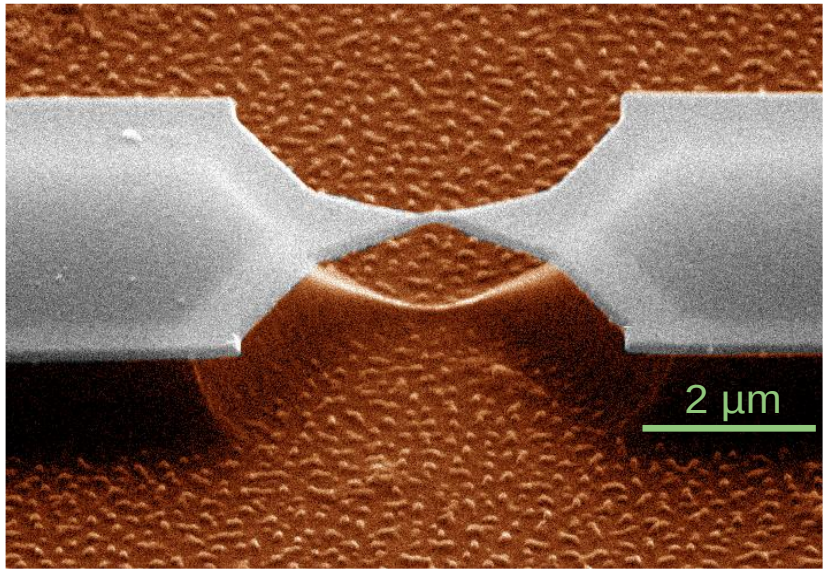
1. Well-controlled system:
   few channels, measurable transmissions
2. Phase biasing
3. Detection of transitions

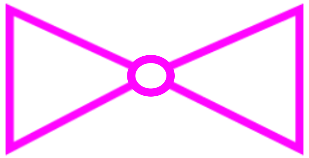


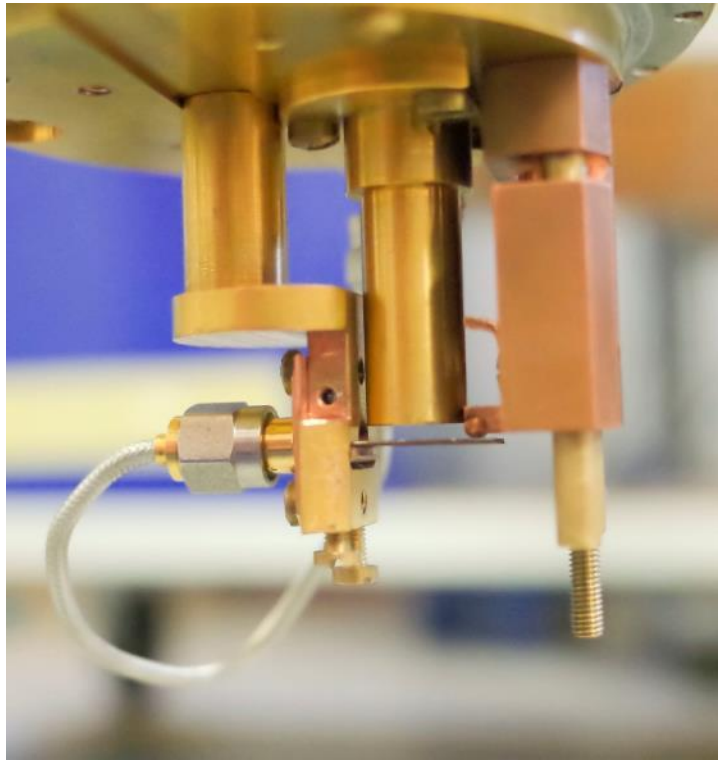
SUPERCONDUCTING ATOMIC CONTACTS



ATOMIC CONTACTS FROM MICROFABRICATED BREAK JUNCTIONS





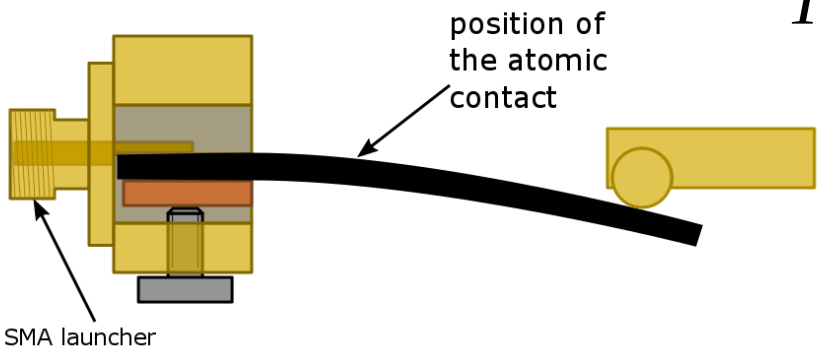


Aluminum thin film

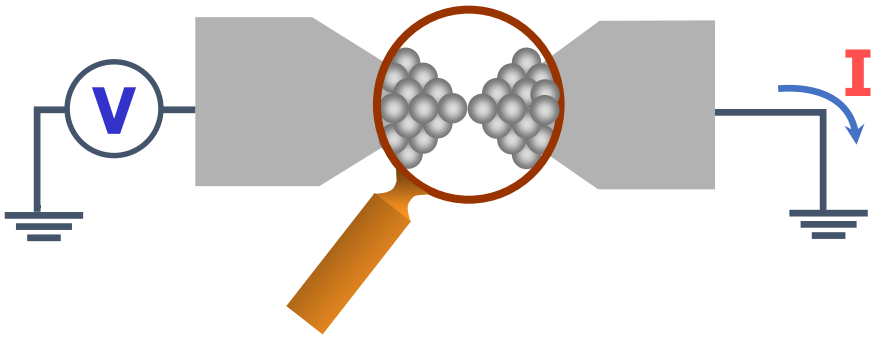
$T_C \sim 1.1\,\mathrm{K}$

$T \sim 20\ \mathrm{mK}$

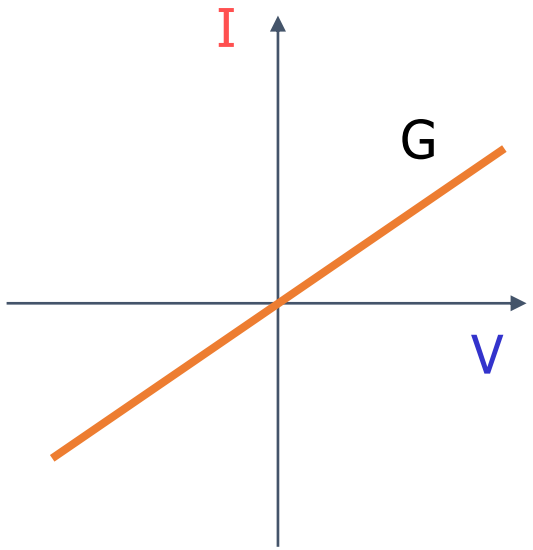
- Few conduction channels (Al: 3)
- Stable
- Mechanical control of $\{\tau_i\}$

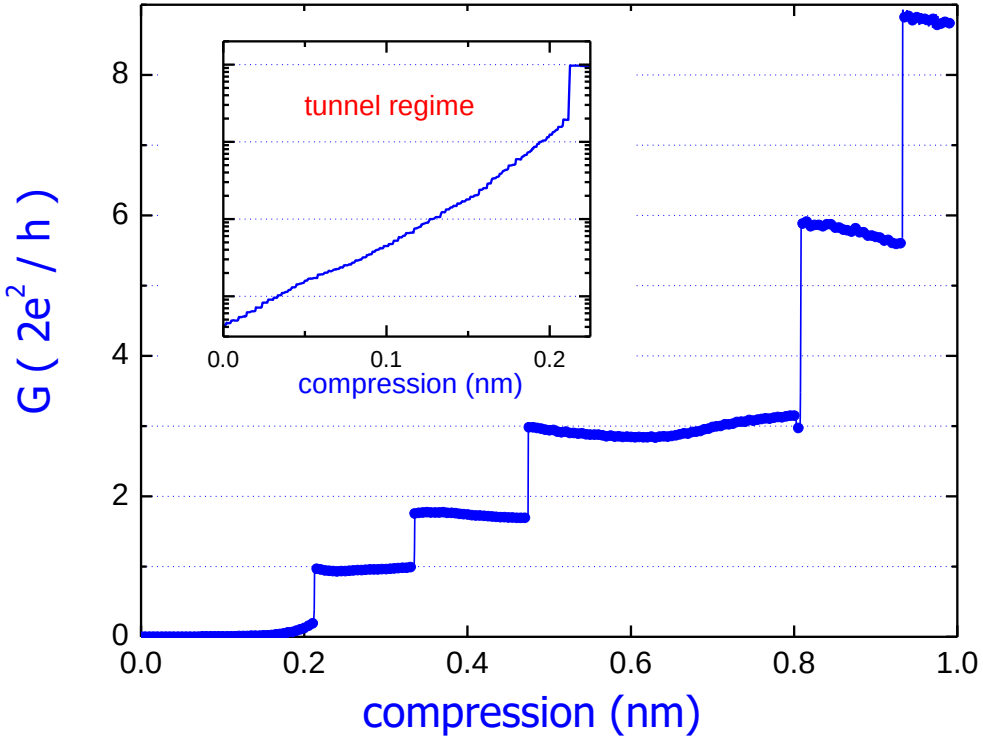


CONDUCTANCE OF ATOMIC CONTACTS IN THE NORMAL STATE

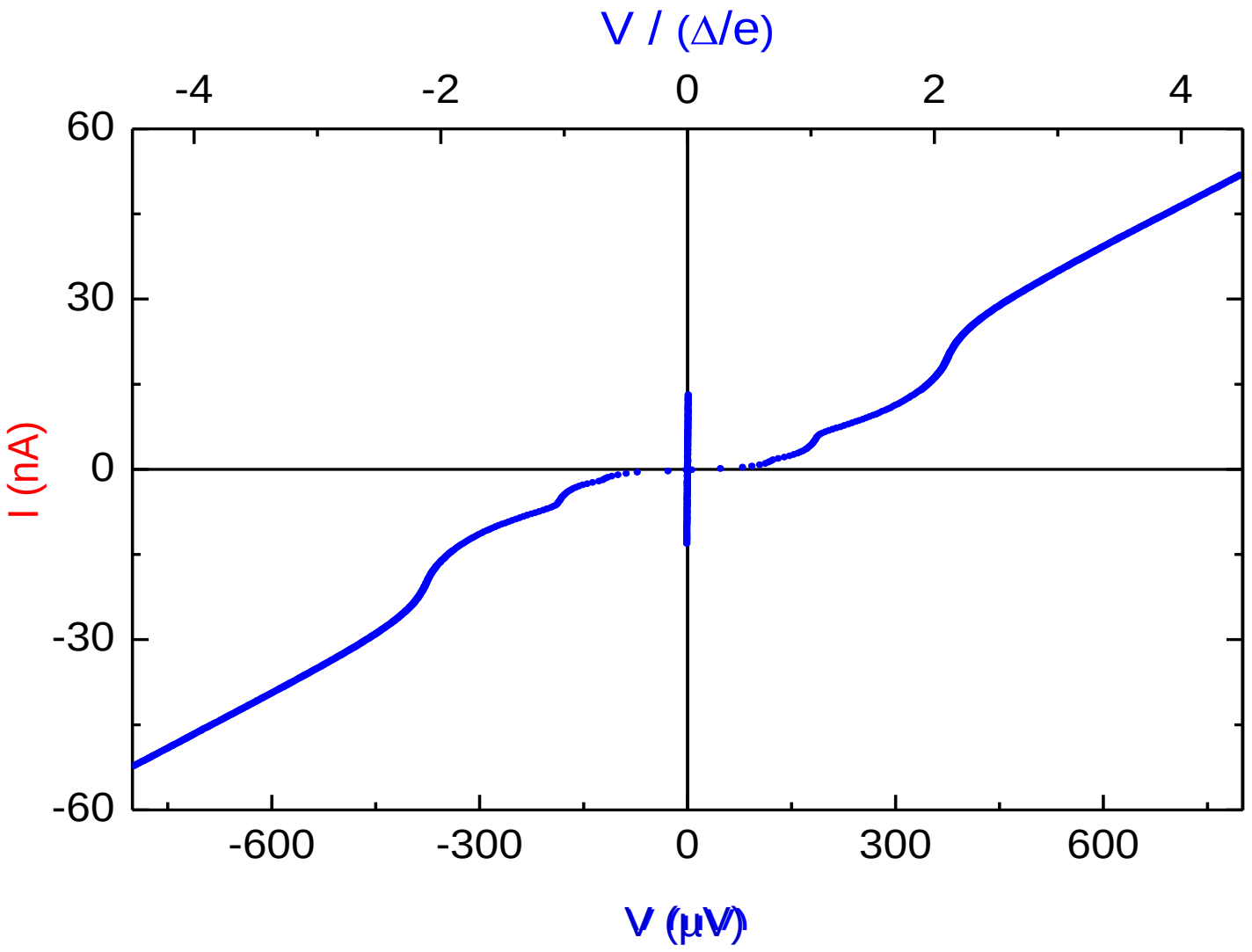


Aluminum
H = 200mT, T = 50 mK

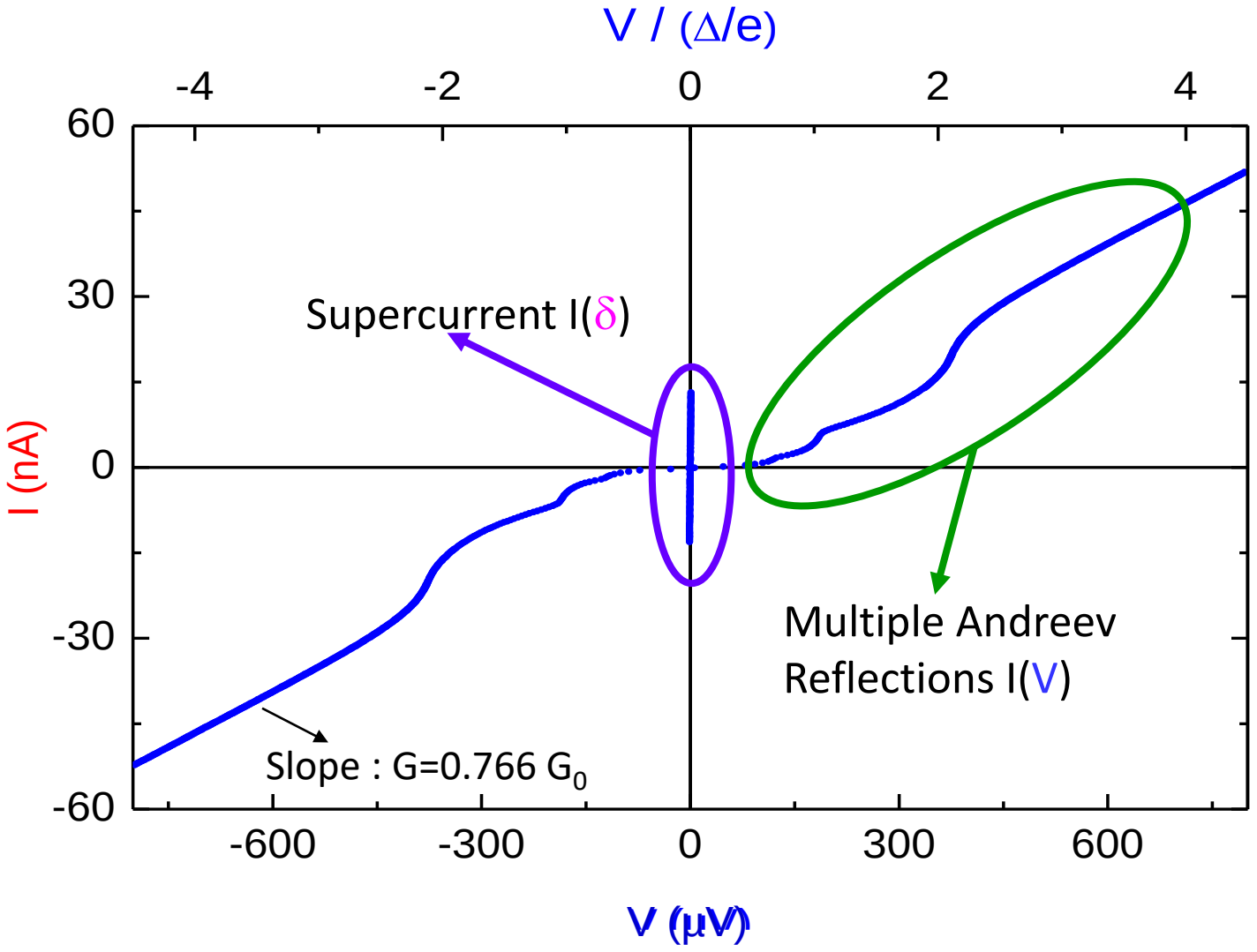




(I-V) OF A ONE-ATOM-CONTACT

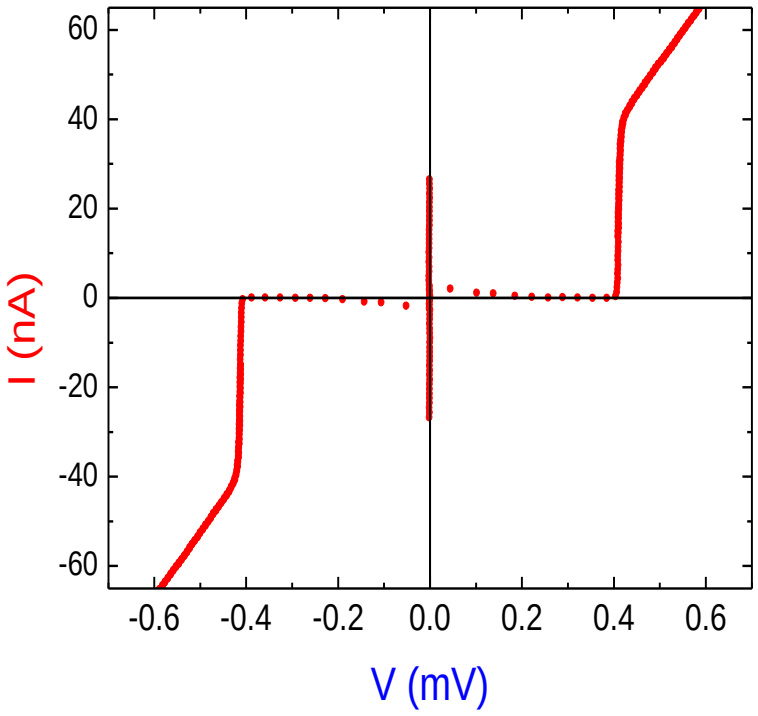


(I-V) OF A ONE-ATOM-CONTACT

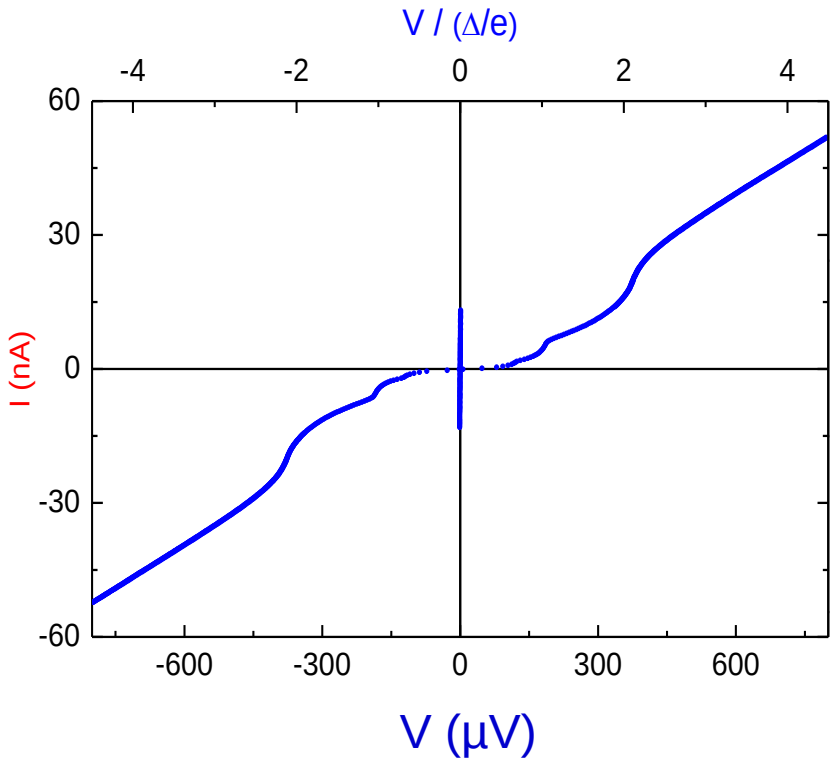


(I-V) OF A ONE-ATOM-CONTACT

TUNNEL JUNCTION

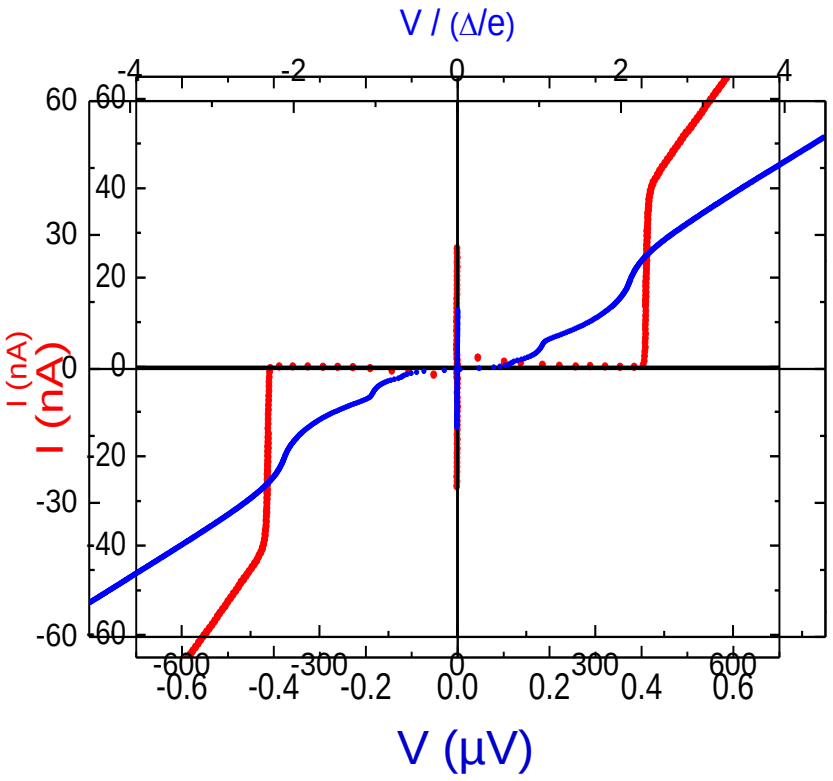


ATOMIC CONTACT



(I-V) OF A ONE-ATOM-CONTACT

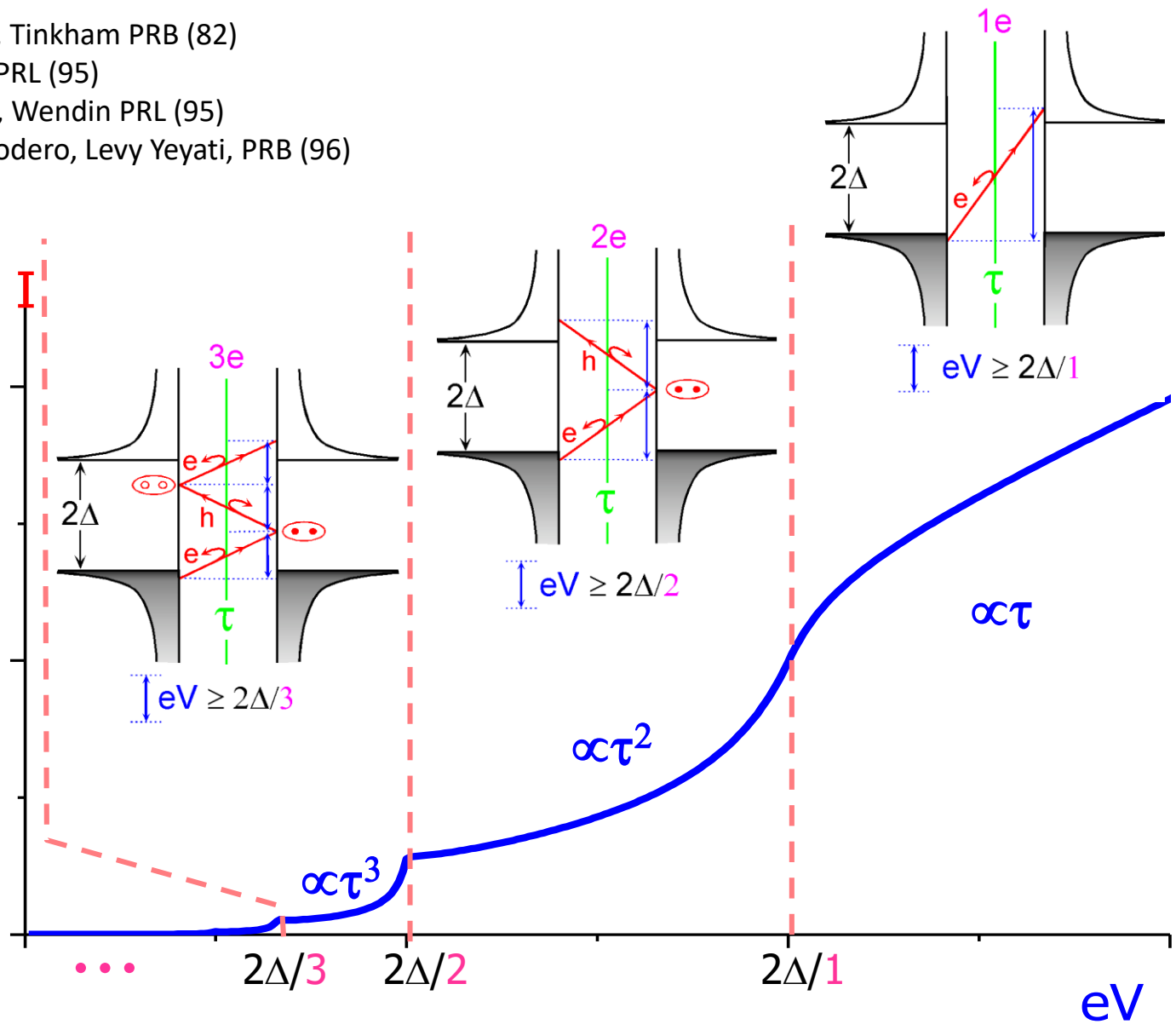
TUNNEL JUNCTION **ATOMIC CONTACT**



MULTIPLE ANDREEV REFLECTIONS

Blonder, Klapwijk, Tinkham PRB (82)
Averin & Bardas, PRL (95)
Bratus ,Shumeiko, Wendin PRL (95)
Cuevas, Martin-Rodero, Levy Yeyati, PRB (96)

MULTIPLE CHARGE TRANSFER PROCESSES



MULTIPLE ANDREEV REFLECTIONS

N. Agraït et al. / Physics Reports 377 (2003) 81–279

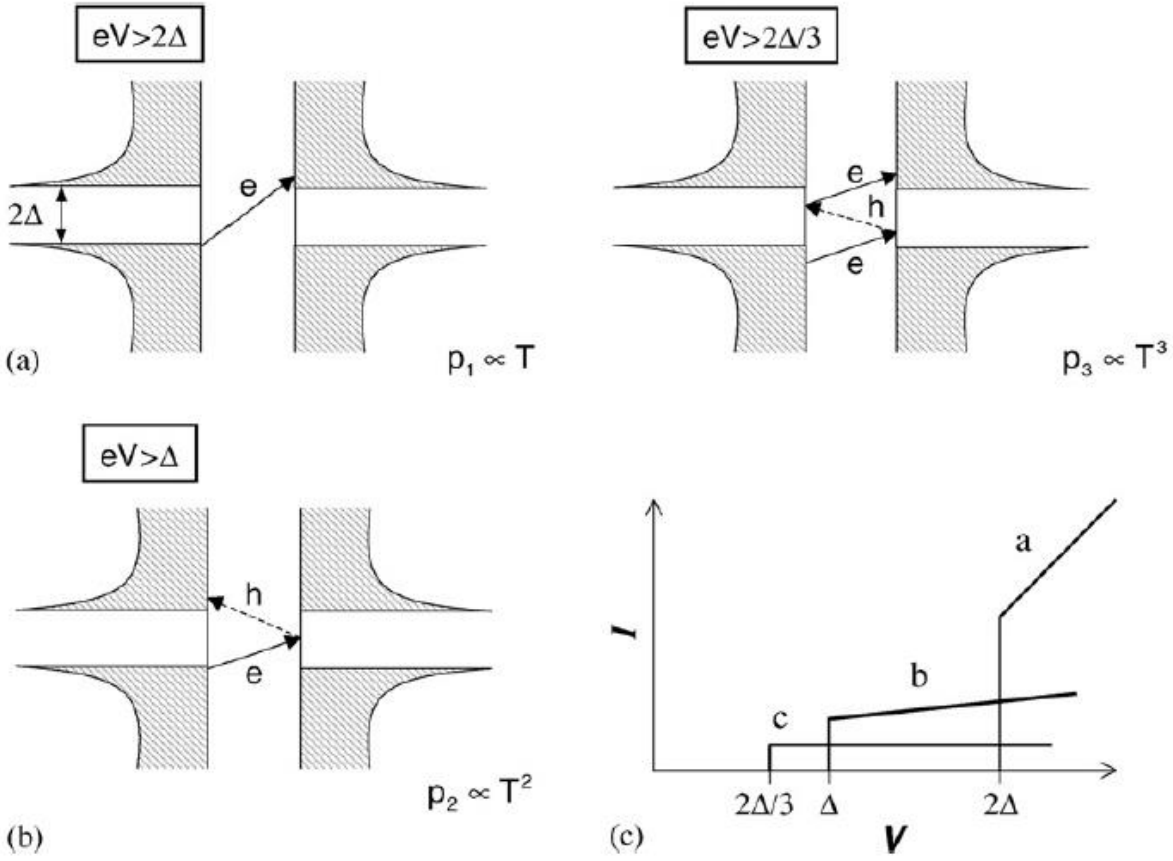
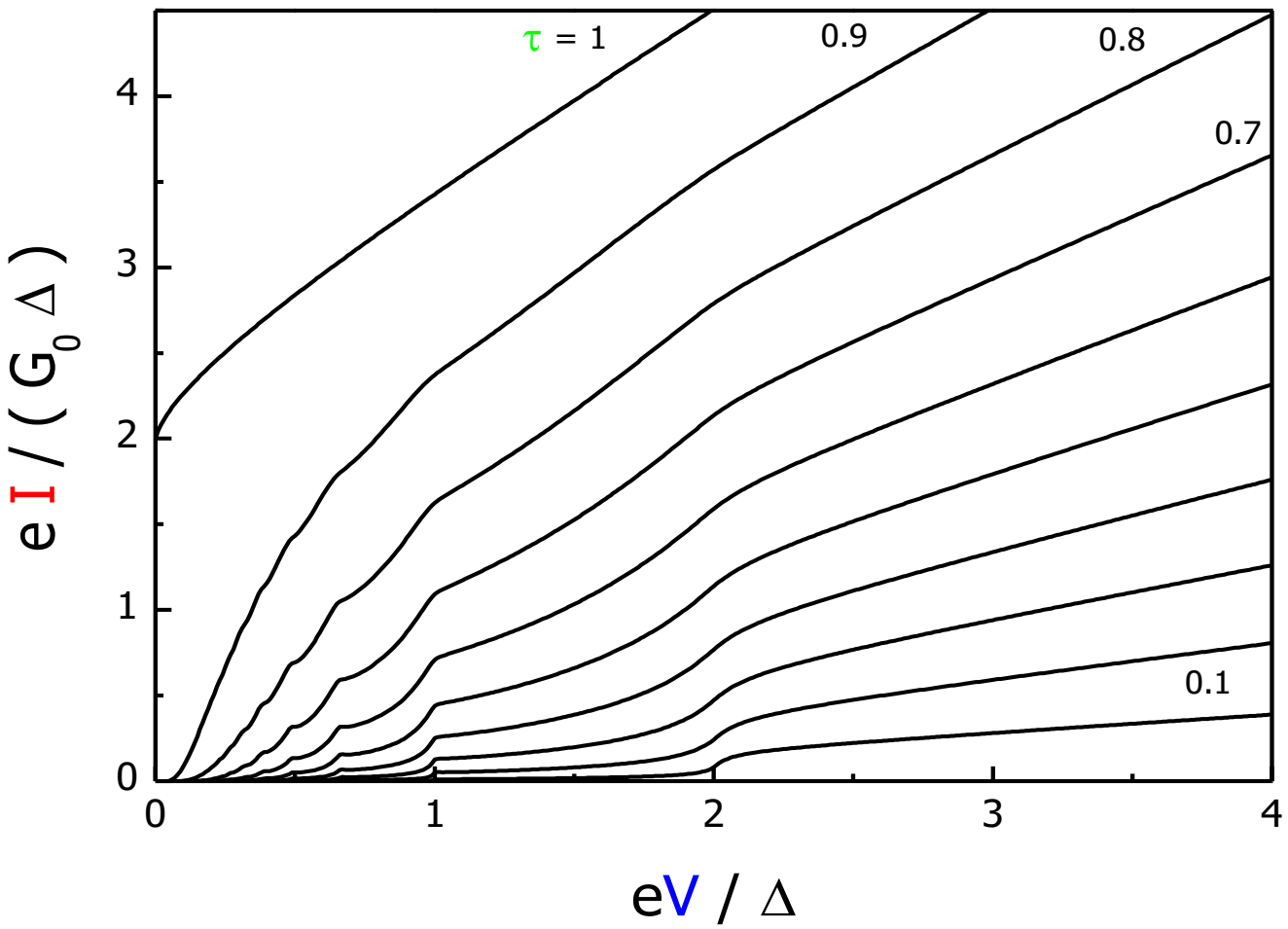


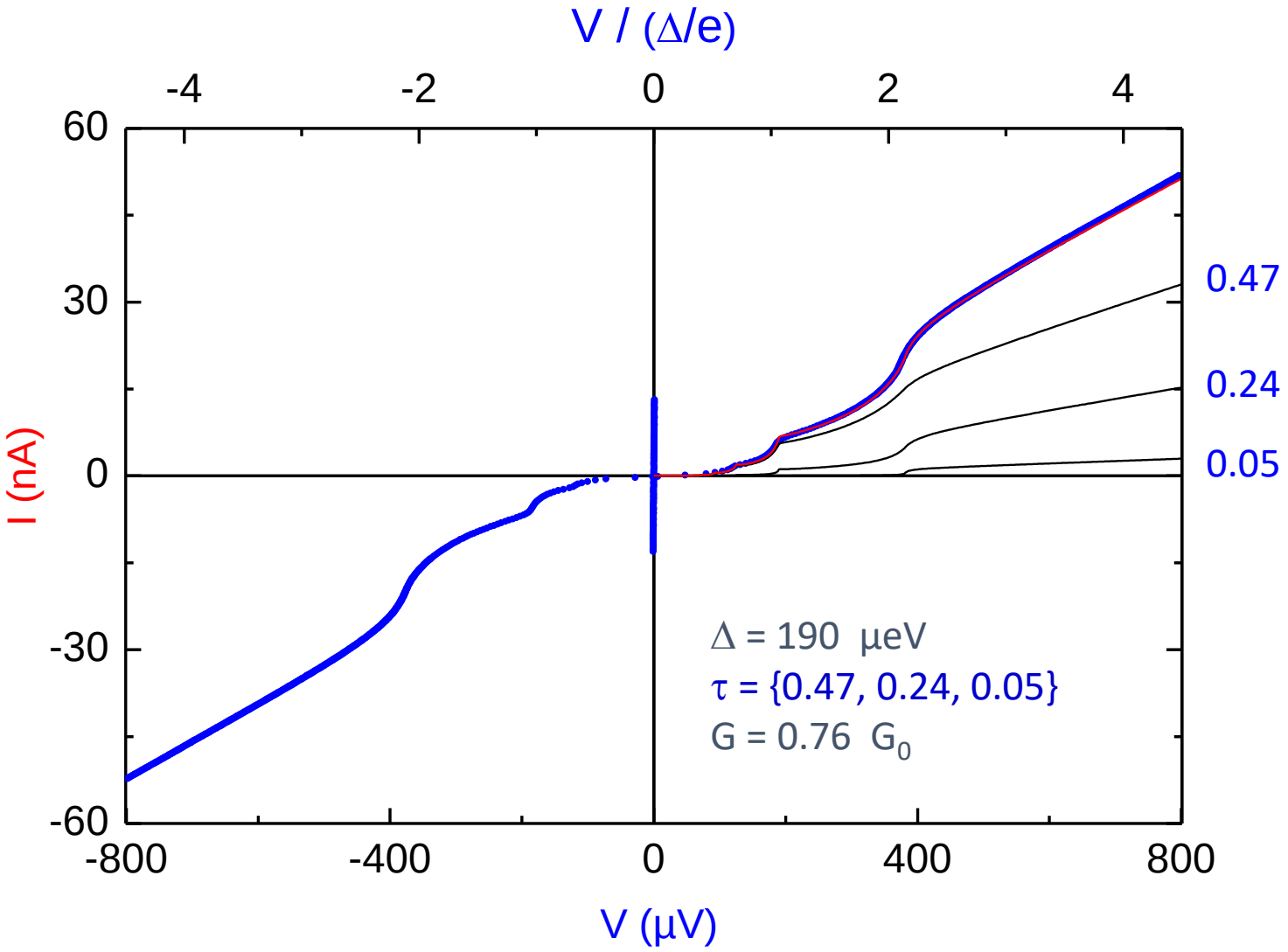
Fig. 23. Schematic explanation of the subgap structure in superconducting contacts.

DC MAR CURRENT



Averin & Bardas, PRL **75**, 1831 (1995)
Cuevas et al., PRB **54**, 7366 (1996)
V. Shumeiko et al., LTP **23**, 181 (1997)

DETERMINATION DES TRANSMISSIONS



I-V characteristic $\Longrightarrow$ $\{\tau_1, \ldots, \tau_N\}$

Scheer *et al.* PRL 1997

DETERMINATION DES TRANSMISSIONS

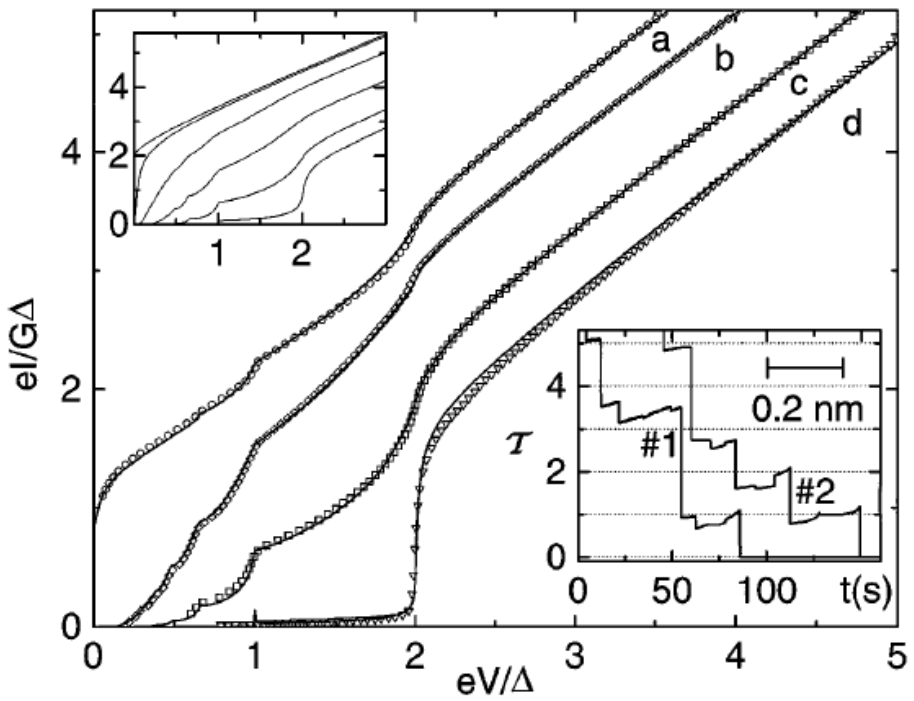


FIG. 1. Measured current-voltage characteristics (symbols) of four different configurations of sample #1 at 30 mK and best numerical fits (lines). The individual channel transmissions and total transmission $\mathcal{T}$ obtained from the fits are (a) $\tau_1 = 0.997$, $\tau_2 = 0.46$, $\tau_3 = 0.29$, $\mathcal{T} = 1.747$; (b) $\tau_1 = 0.74$, $\tau_2 = 0.11$, $\mathcal{T} = 0.85$; (c) $\tau_1 = 0.46$, $\tau_2 = 0.35$, $\tau_3 = 0.07$, $\mathcal{T} = 0.88$; and (d) $\mathcal{T} = \tau_1 = 0.025$. Voltage and cur-

I-V characteristic $\Longrightarrow$ $\{\tau_1, \ldots, \tau_N\}$

Scheer *et al.* PRL 1997

EFFECTIVE CHARGE TRANSFER

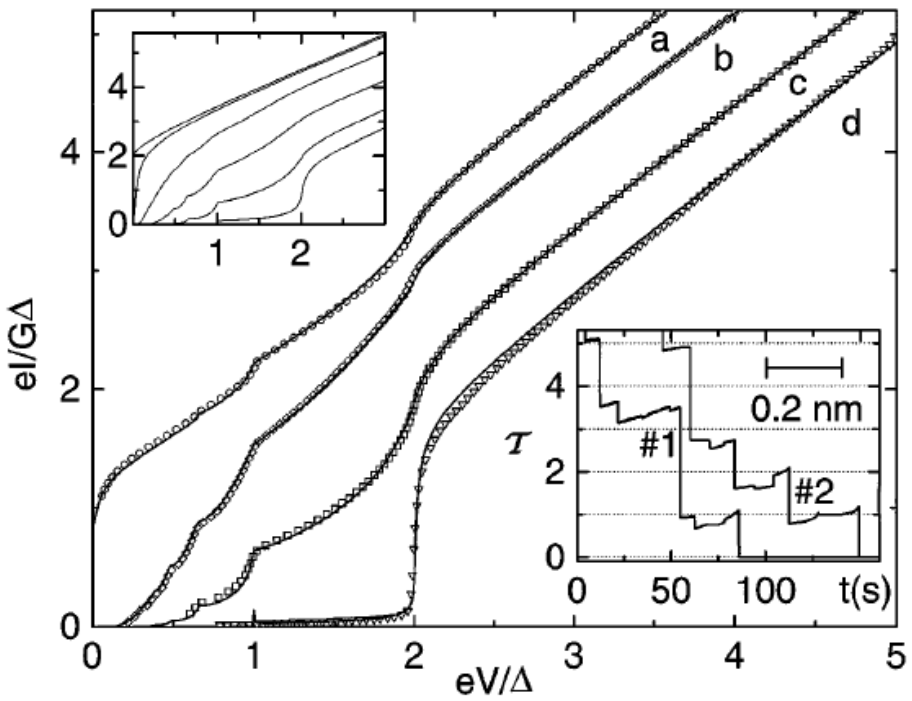


FIG. 1. Measured current-voltage characteristics (symbols) of four different configurations of sample #1 at 30 mK and best numerical fits (lines). The individual channel transmissions and total transmission $\mathcal{T}$ obtained from the fits are (a) $\tau_1 = 0.997$, $\tau_2 = 0.46$, $\tau_3 = 0.29$, $\mathcal{T} = 1.747$; (b) $\tau_1 = 0.74$, $\tau_2 = 0.11$, $\mathcal{T} = 0.85$; (c) $\tau_1 = 0.46$, $\tau_2 = 0.35$, $\tau_3 = 0.07$, $\mathcal{T} = 0.88$; and (d) $\mathcal{T} = \tau_1 = 0.025$. Voltage and cur-

Scheer *et al.* PRL 1997

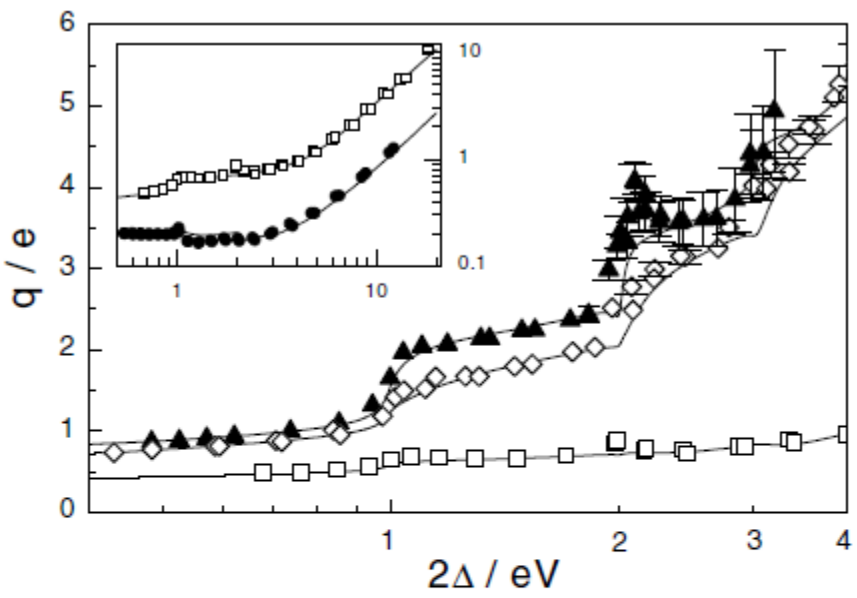
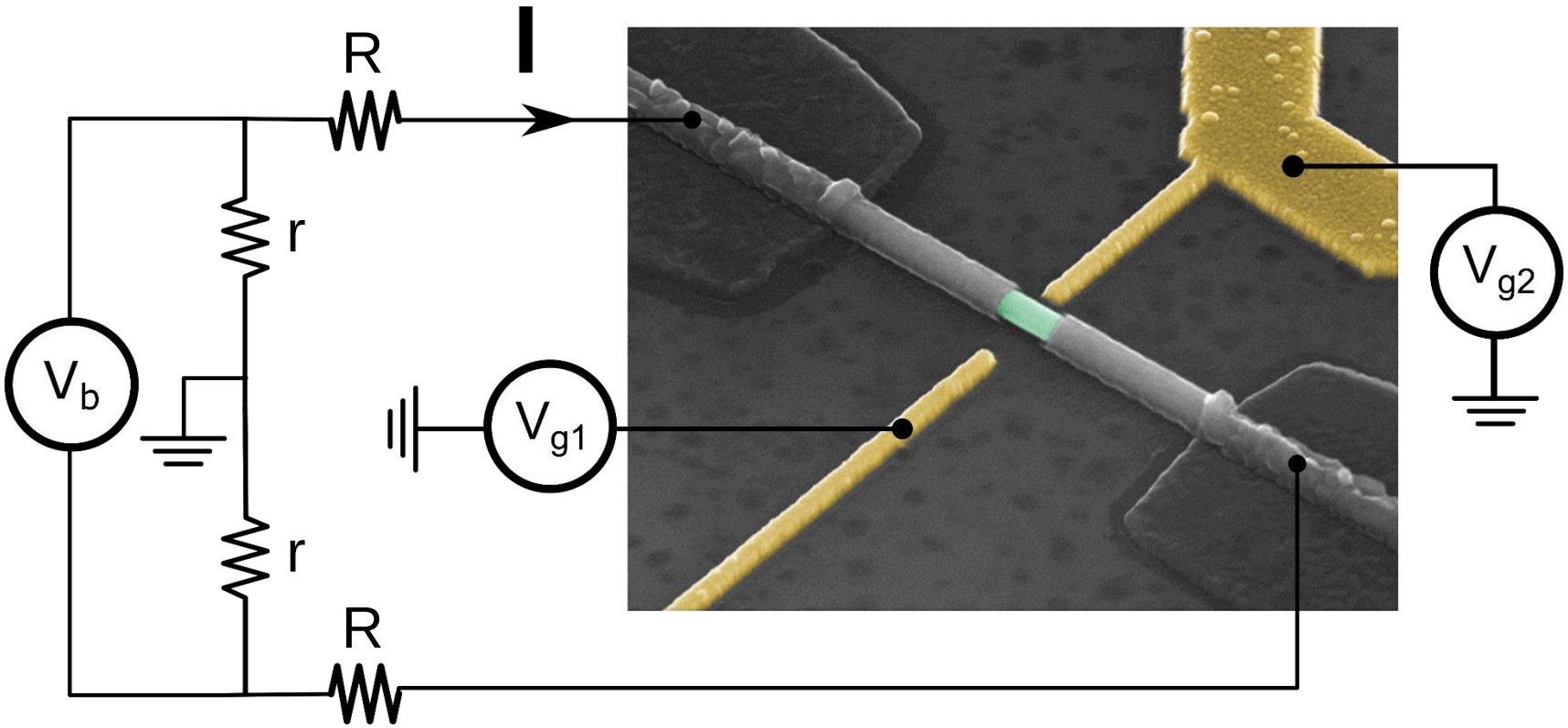


FIG. 5. Symbols: effective size $q/e = S_I/2eI$ of the noise shots versus reduced inverse voltage, for three different contacts in the superconducting state. These symbols are experimental results and the lines are predictions of the MAR theory for noise, using the mesoscopic pin codes determined from fits of the $IV's$. From top to bottom: $\{0.40, 0.27, 0.03\}$, $\{0.68, 0.25, 0.22\}$, $\{0.98, 0.55, 0.24, 0.22\}$. Inset: data for two contacts containing an almost ballistic channel (top $\{0.98, 0.55, 0.24, 0.22\}$, bottom $\{0.996, 0.26\}$) shown on a larger scale.

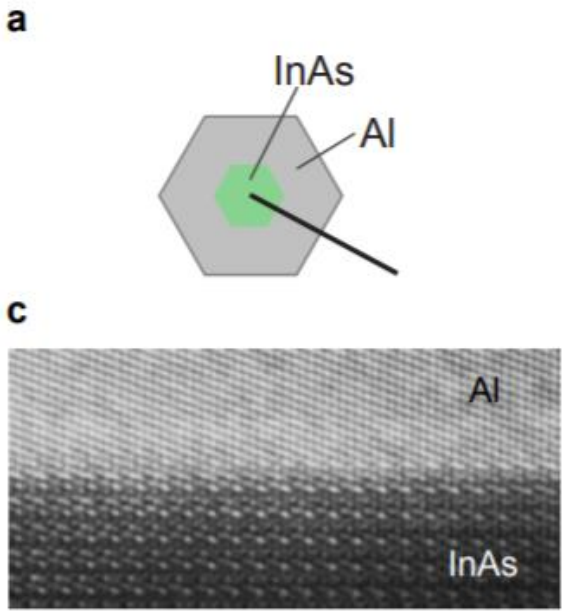
Cron *et al.*, PRL (2001)

I-V characteristic $\Longrightarrow$ $\{\tau_1, \ldots, \tau_N\}$

epiAl-InAs NANOWIRES

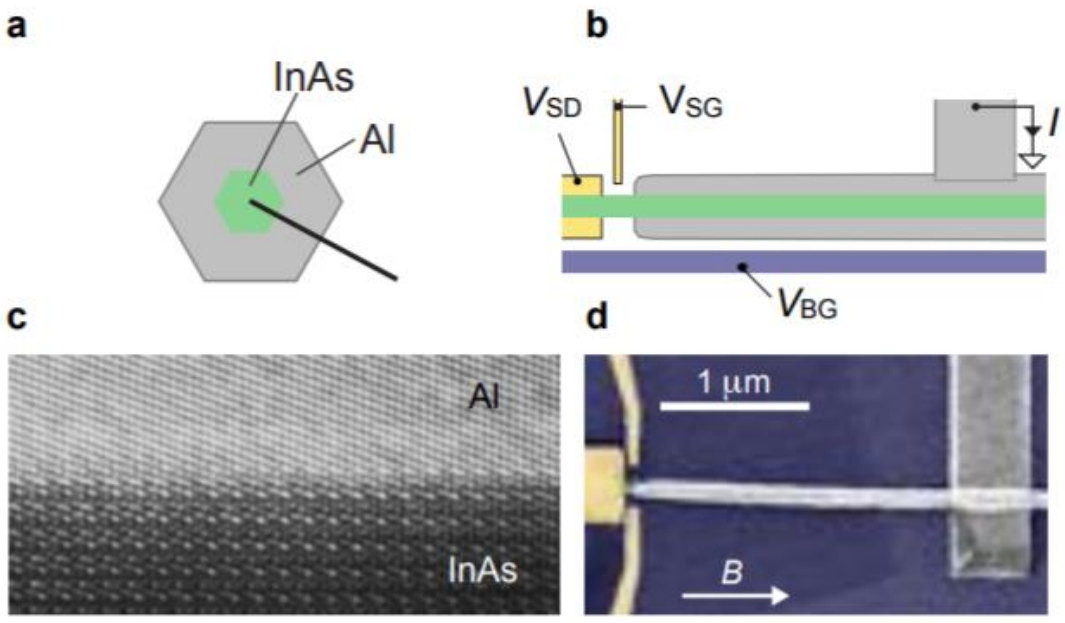


epiAl-InAs NANOWIRES



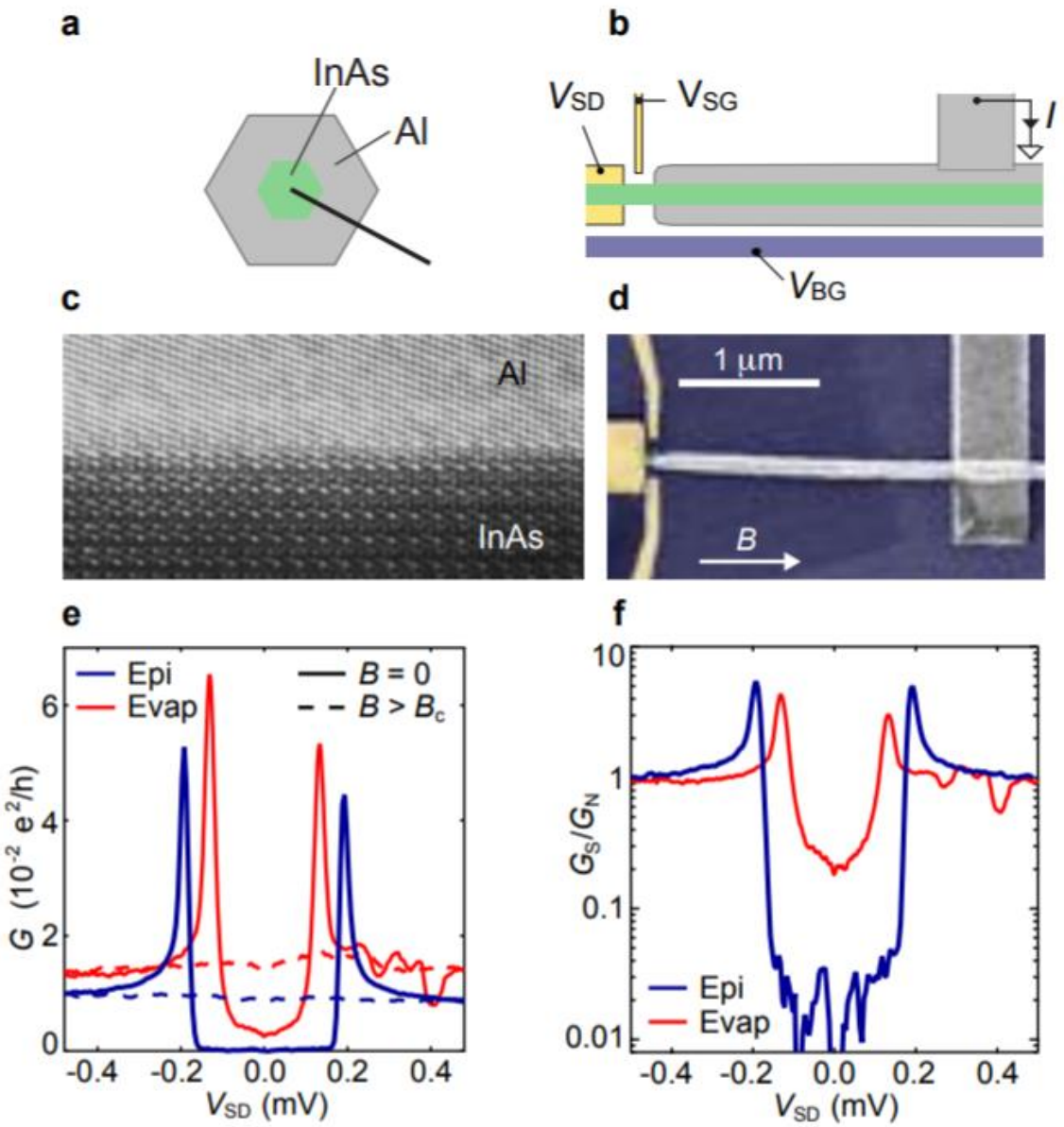
Wang *et al.,* Nature (2015)

epiAl-InAs NANOWIRES



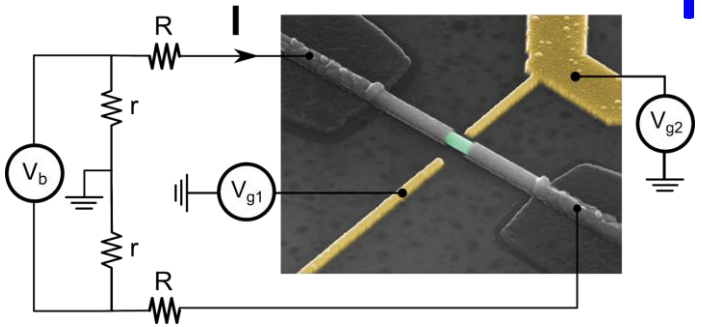
Wang *et al.,* Nature (2015)

epiAl-InAs NANOWIRES

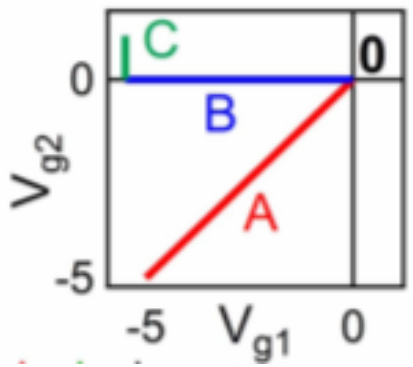


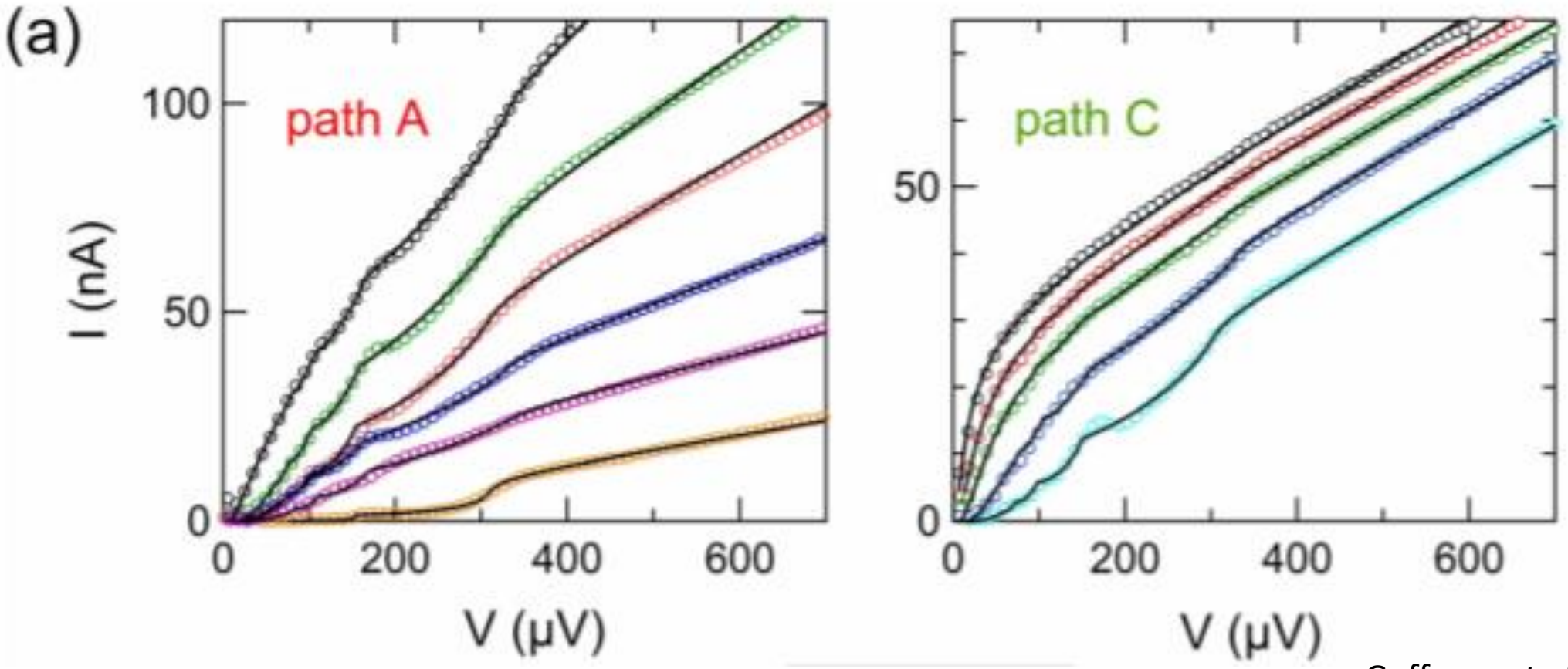
Chang *et al.*, Nature (2015)

MAR in epiAl-InAs NANOWIRES



Gating path

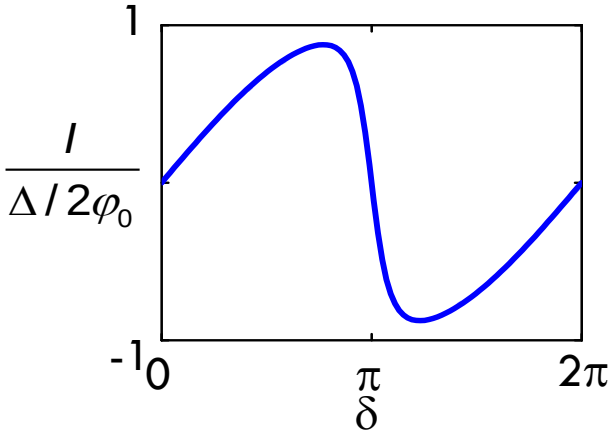




Goffman *et al.*, NJP (2017)

PROBE MESOSCOPIC THEORY OF JOSEPHSON EFFECT

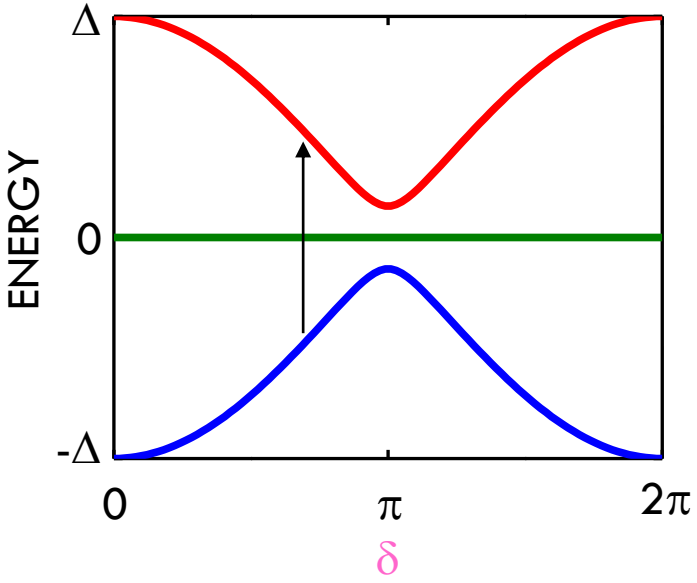
- **probe** $I(\delta)$ in $|g\rangle$



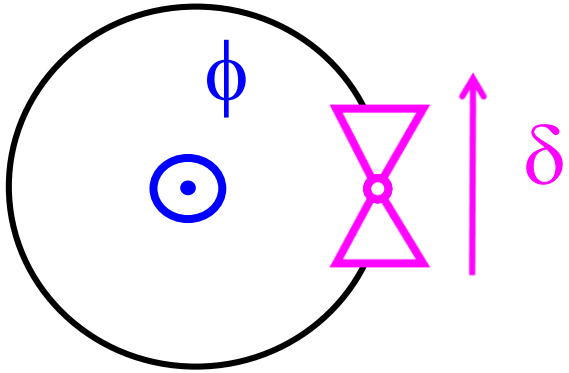
- Spectroscopy of ABS
- Drive coherent oscillations between states

➔ requires:

1. Well-controlled system:
   few channels, measurable transmissions
2. Phase biasing
3. Detection of transitions



PHASE BIASING

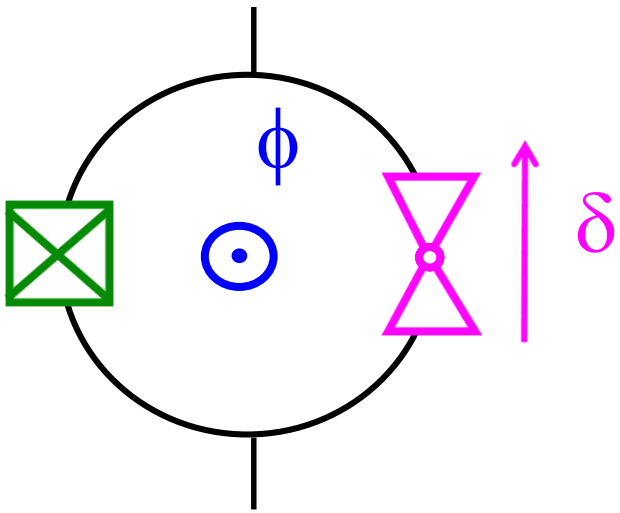


TO PHASE BIAS ENCLOSE IN
A SUPERCONDUCTING LOOP
THREADED BY A FLUX

$$\delta = 2\pi \frac{\phi}{\phi_0}$$

PHASE AND VOLTAGE BIASING

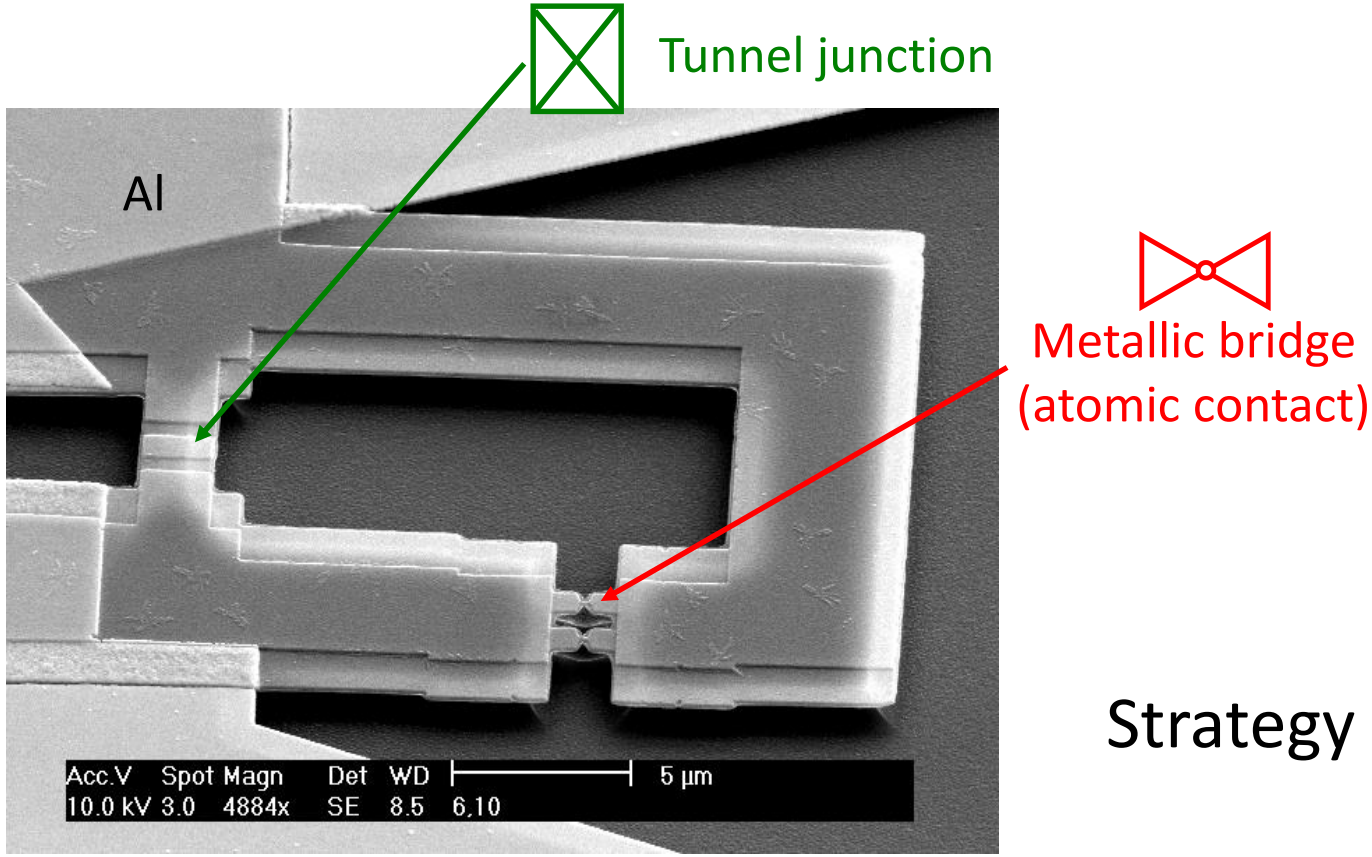
AN ANCILLARY JUNCTION
ACTS AS A SUPERCONDUCTING
REVERSIBLE SWITCH

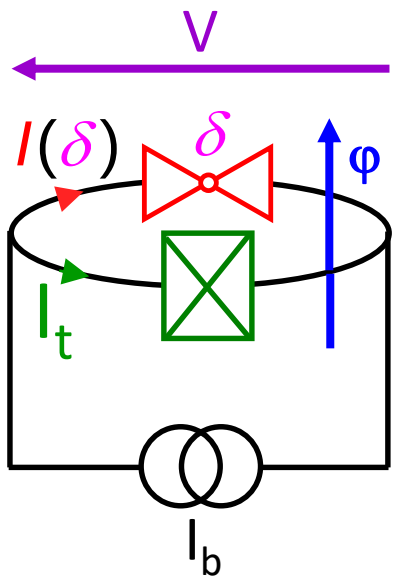


when biased with a current smaller than critical current of junction, acts as a short
PHASE BIAS of contact

when biased with a current larger than critical current of junction
VOLTAGE BIAS of contact

MEASUREMENT OF CURRENT-PHASE RELATION





$$30\ \text{nA} \approx I_{\max} \ll I_t^0 \approx 0.7\ \mu\text{A}$$

Strategy :

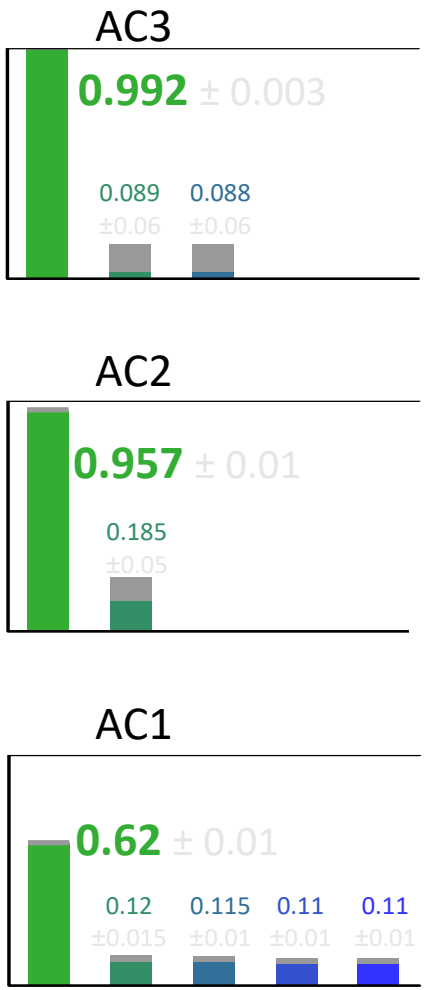
1) Measure $\{\tau_1, \ldots, \tau_M\}$
V>0

2) Measure $I(\delta)$
V=0

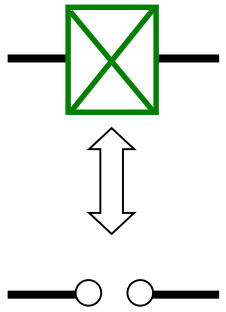
MEASUREMENT OF TRANSMISSIONS $\{\tau_1,\ldots,\tau_M\}$

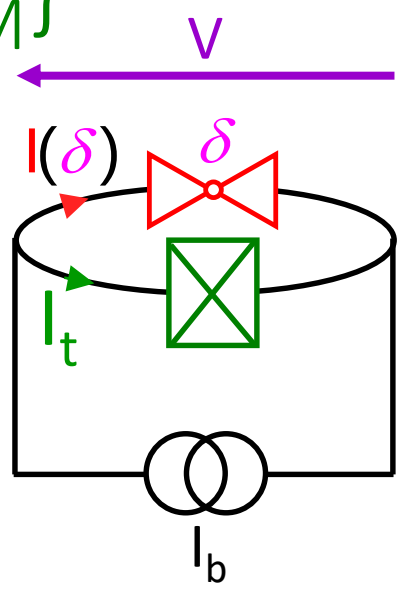
method: Scheer *et al.* 1997

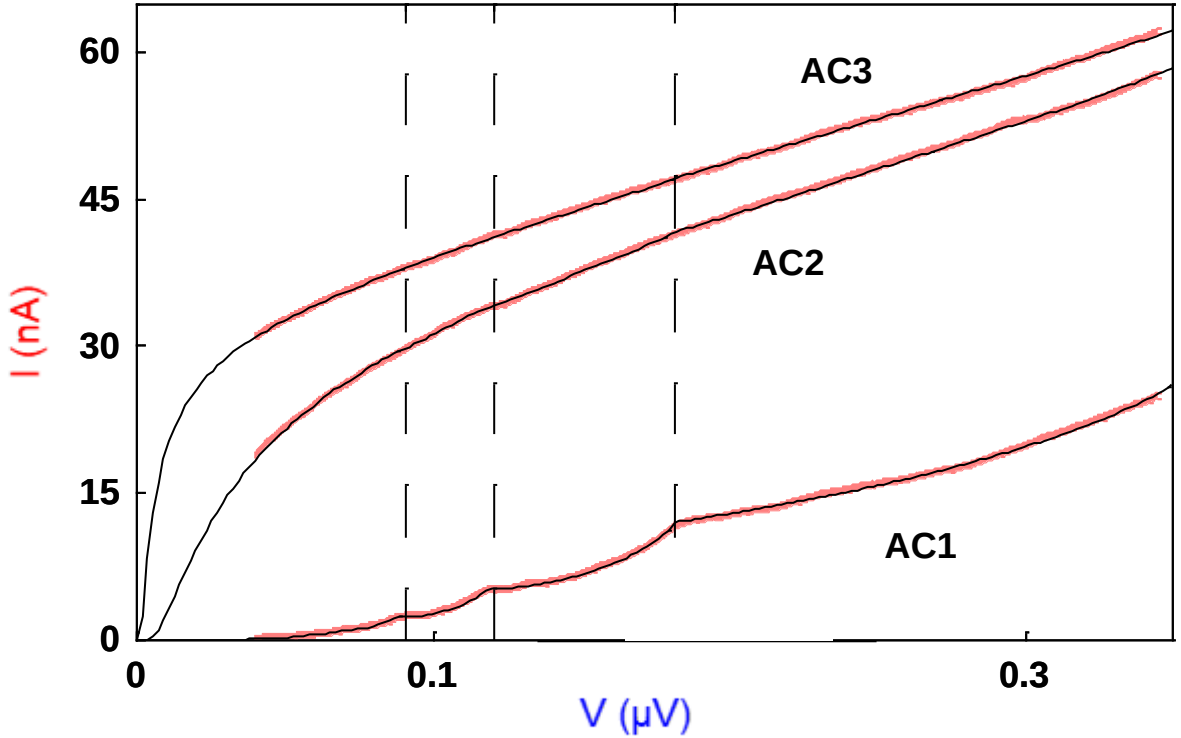
Transmissions



Measure I(V)



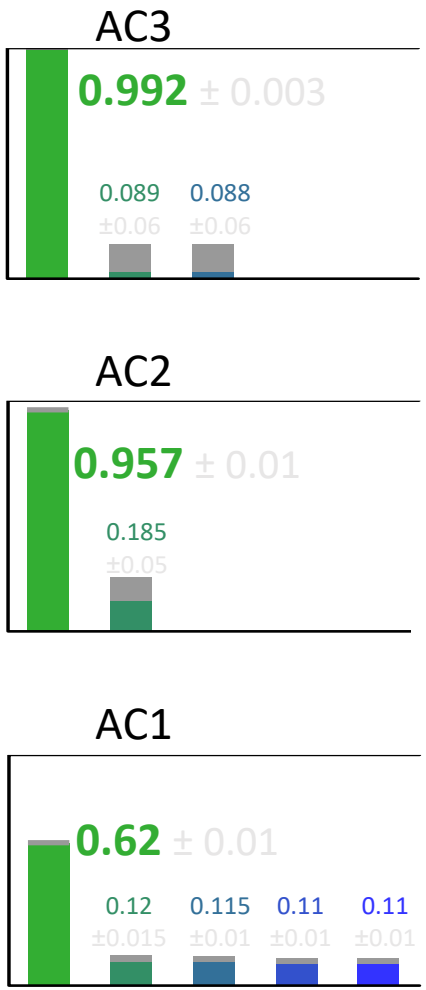




MEASUREMENT OF CURRENT-PHASE RELATION

method: Scheer *et al.* 1997

Transmissions



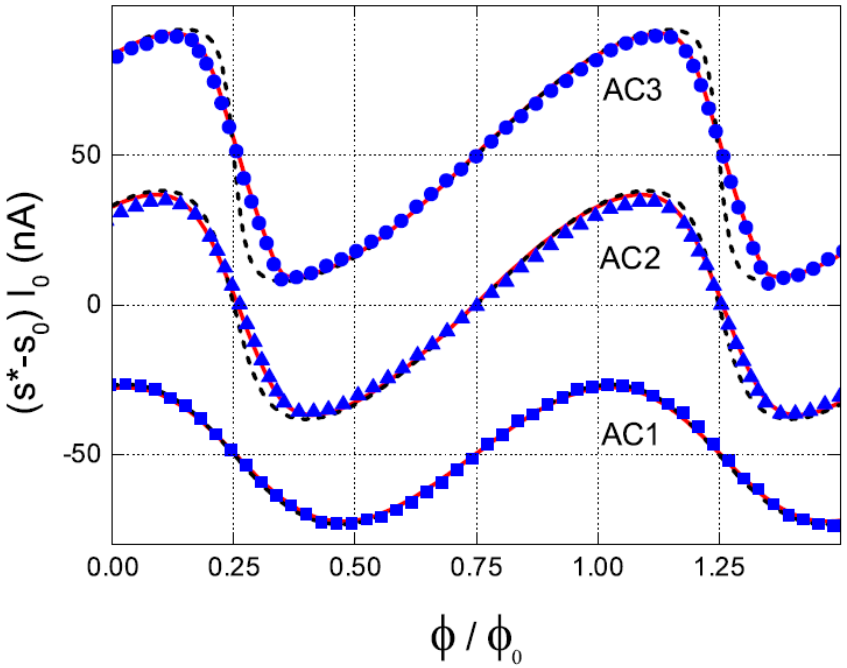


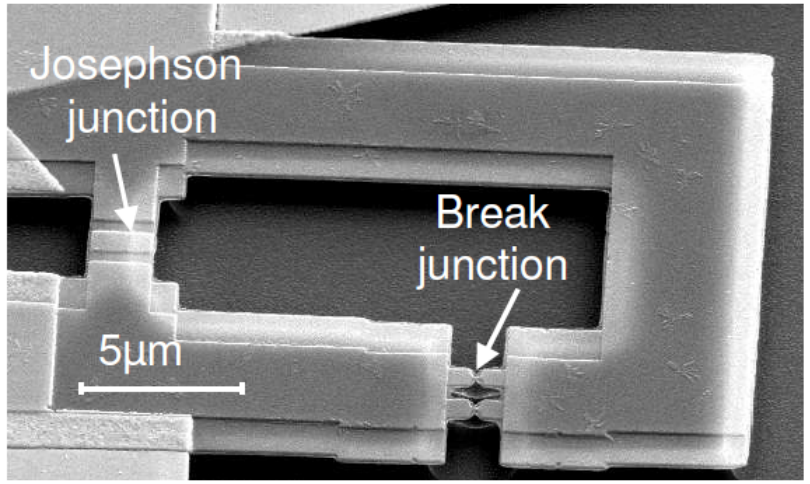
FIG. 3 (color online). Symbols: measured switching current $[s^*(\varphi) - s_0]I_0$ as a function of applied flux ϕ/ϕ_0, for the three SQUIDS corresponding to the contacts of Fig. 2. Curves AC3 and AC1 shifted for clarity. Dashed curves: predicted ground state current-phase relation $I^-_{\{\tau_i\}}(\delta)$. Full lines: predictions of resistively shunted SQUID theory at $T_{esc} = 130$ mK on the basis of Eq. (1). The transmission sets indicated in Fig. 2 caption have been used for both theories.

Della Rocca *et al.*, PRL 2007

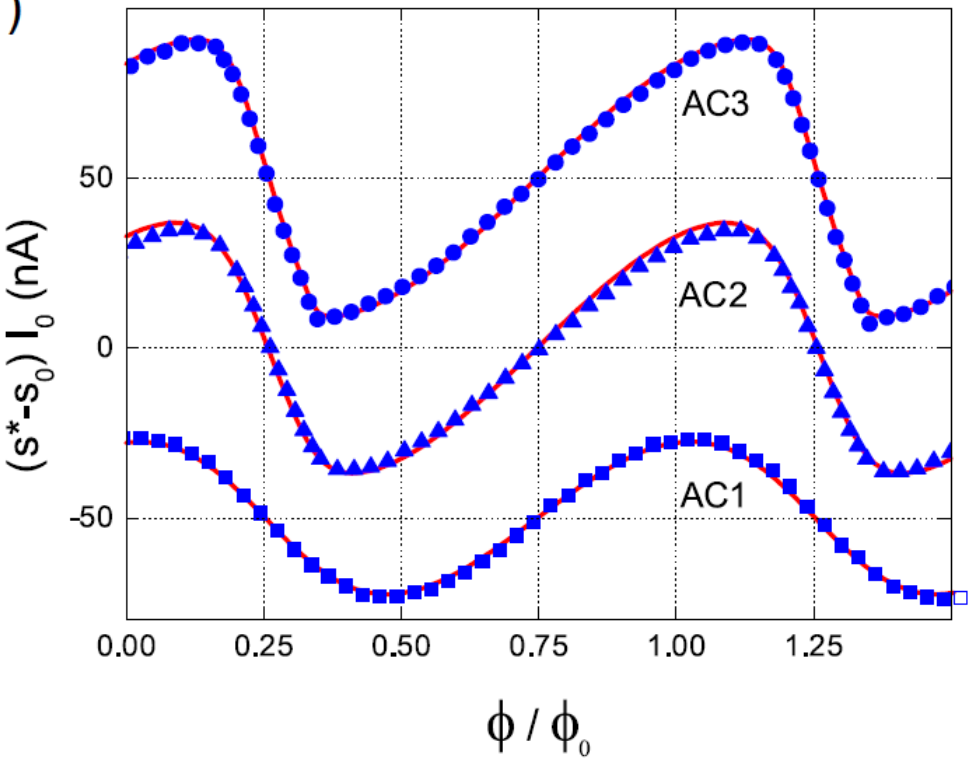
ATOMIC SQUID

Current-phase relation of atomic contacts

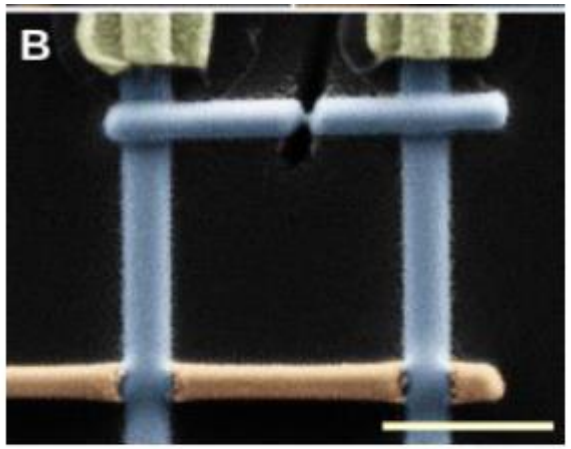
M. Della Rocca, et al. PRL 99 (2007)

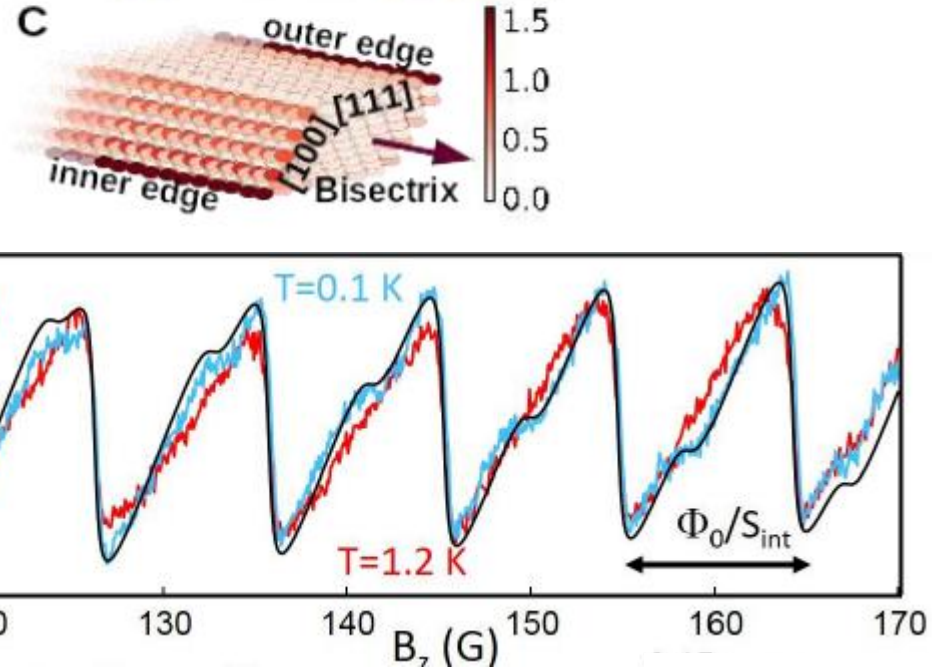


I-V Characteristics → $\{T_1, T_2, T_3\}$



CURRENT-PHASE RELATION AT τ=1 FOR Bi NANOWIRES

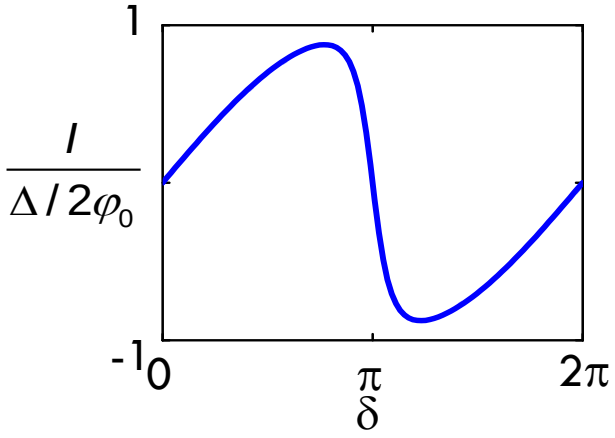




A. Murani et al., arXiv:1609.04848

PROBE MESOSCOPIC THEORY OF JOSEPHSON EFFECT

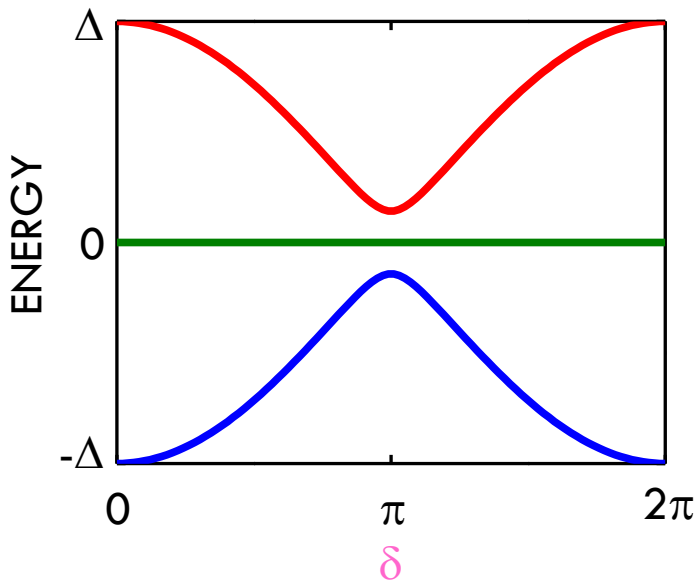
- probe $I(\delta)$ in $|g\rangle$



- **Spectroscopy of ABS**
- Drive coherent oscillations between states

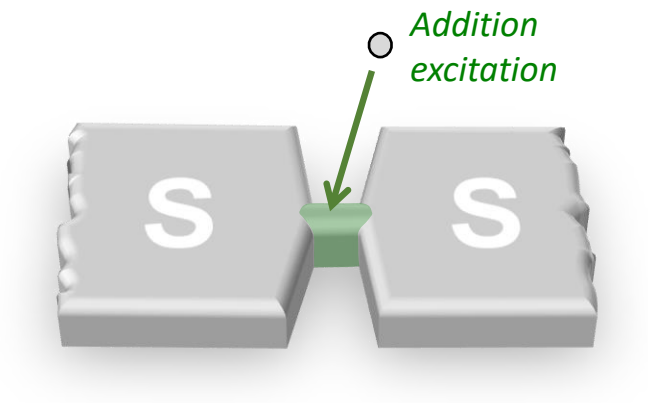
➔ requires:

1. Well-controlled system:
   few channels, measurable transmissions
2. Phase biasing
3. Detection of transitions

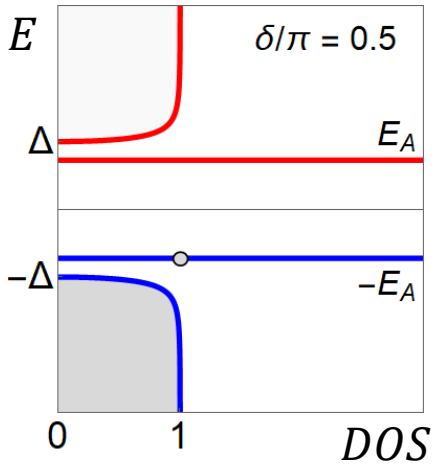


MICROSCOPIC EXCITATIONS IN A SC WEAK-LINK

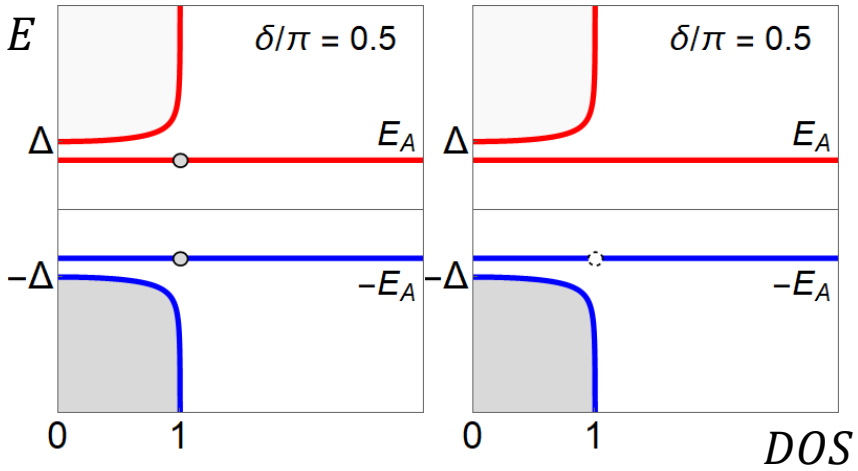
$$\delta \equiv \varphi_L - \varphi_R \neq 0$$



GND State



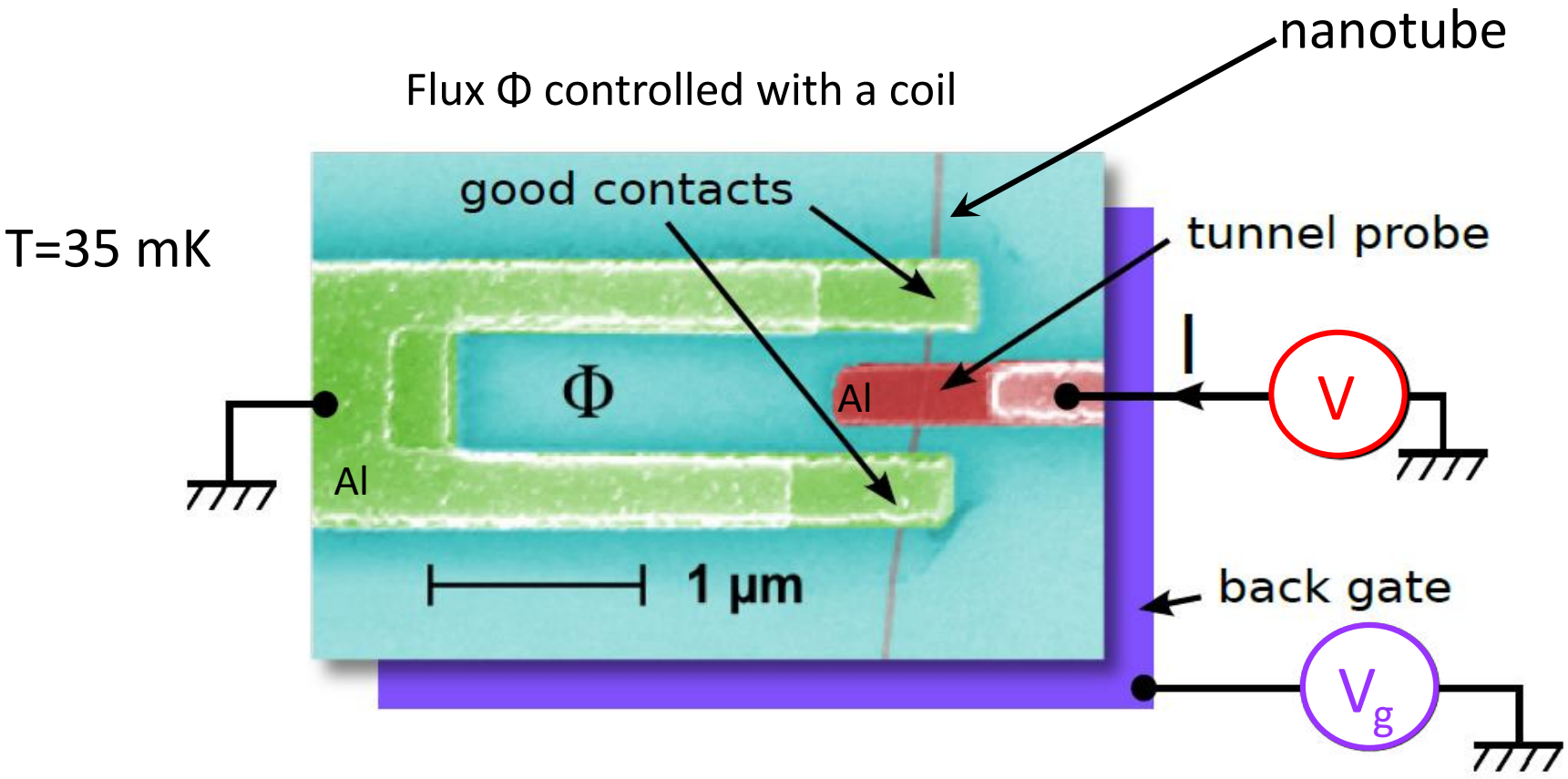
Excited Odd states



Add a quasiparticle

$$E = E_A$$

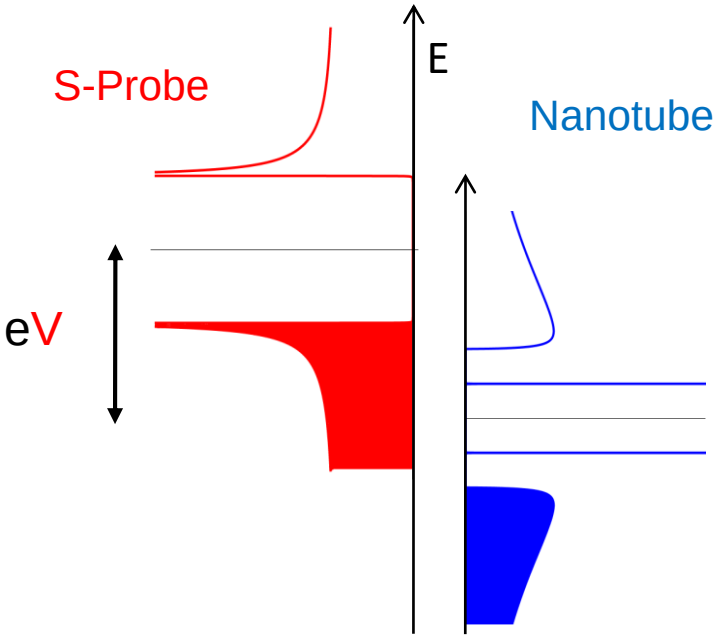
TUNNELING SPECTROSCOPY IN CNT



J.-D. Pillet *et al*, Nature Physics (2010)

TUNNEL CURRENT

$$I(V) \propto \int \left(f_P(\varepsilon - eV) - f_{NT}(\varepsilon) \right) \rho_{NT}(\varepsilon)\, \rho_P(\varepsilon - eV)\, d\varepsilon$$

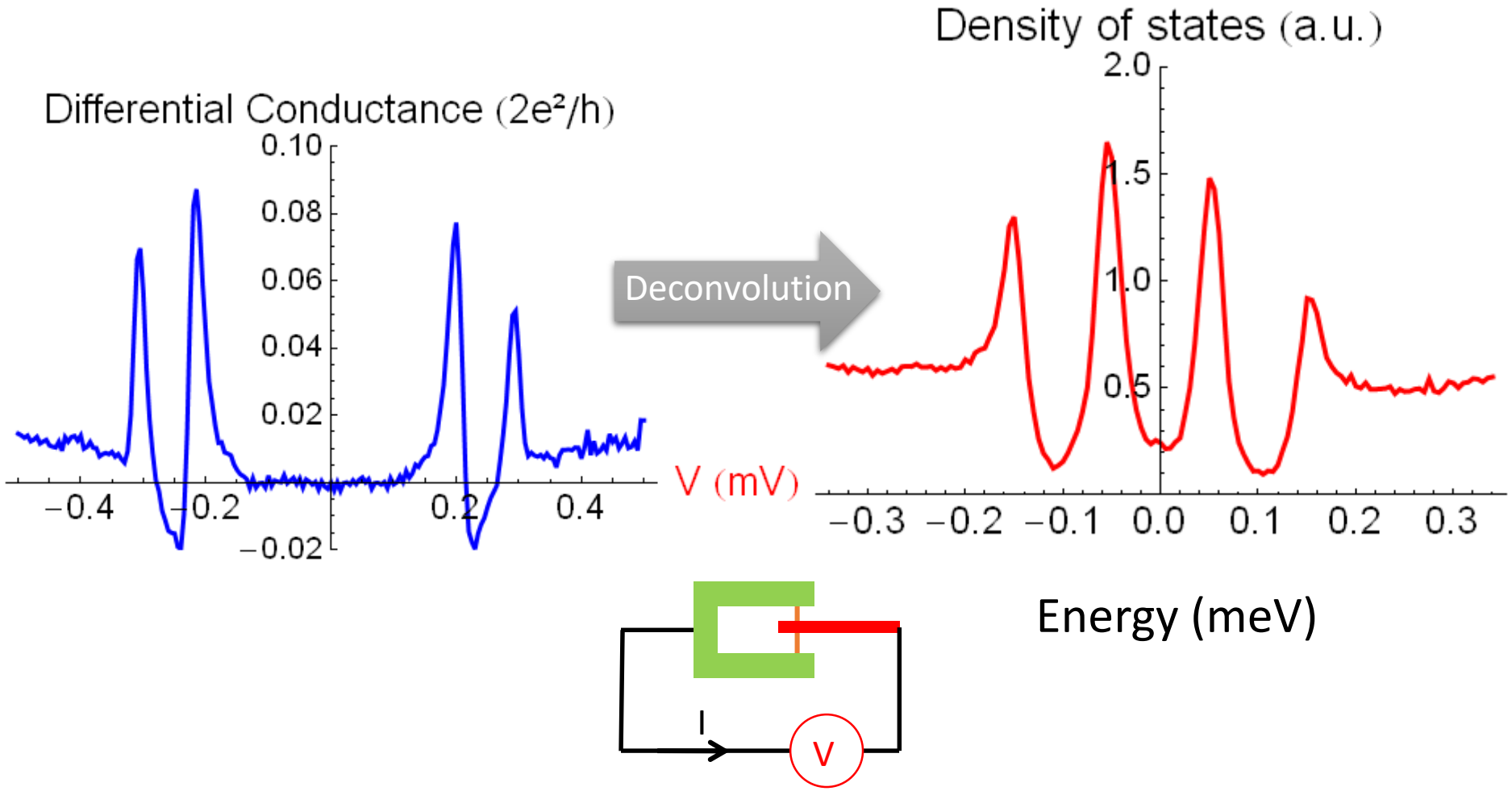


Differential conductance

$$\boxed{\frac{\partial I}{\partial V}(V) \propto g(\varepsilon, V) \otimes \rho_{NT}(\varepsilon)}$$

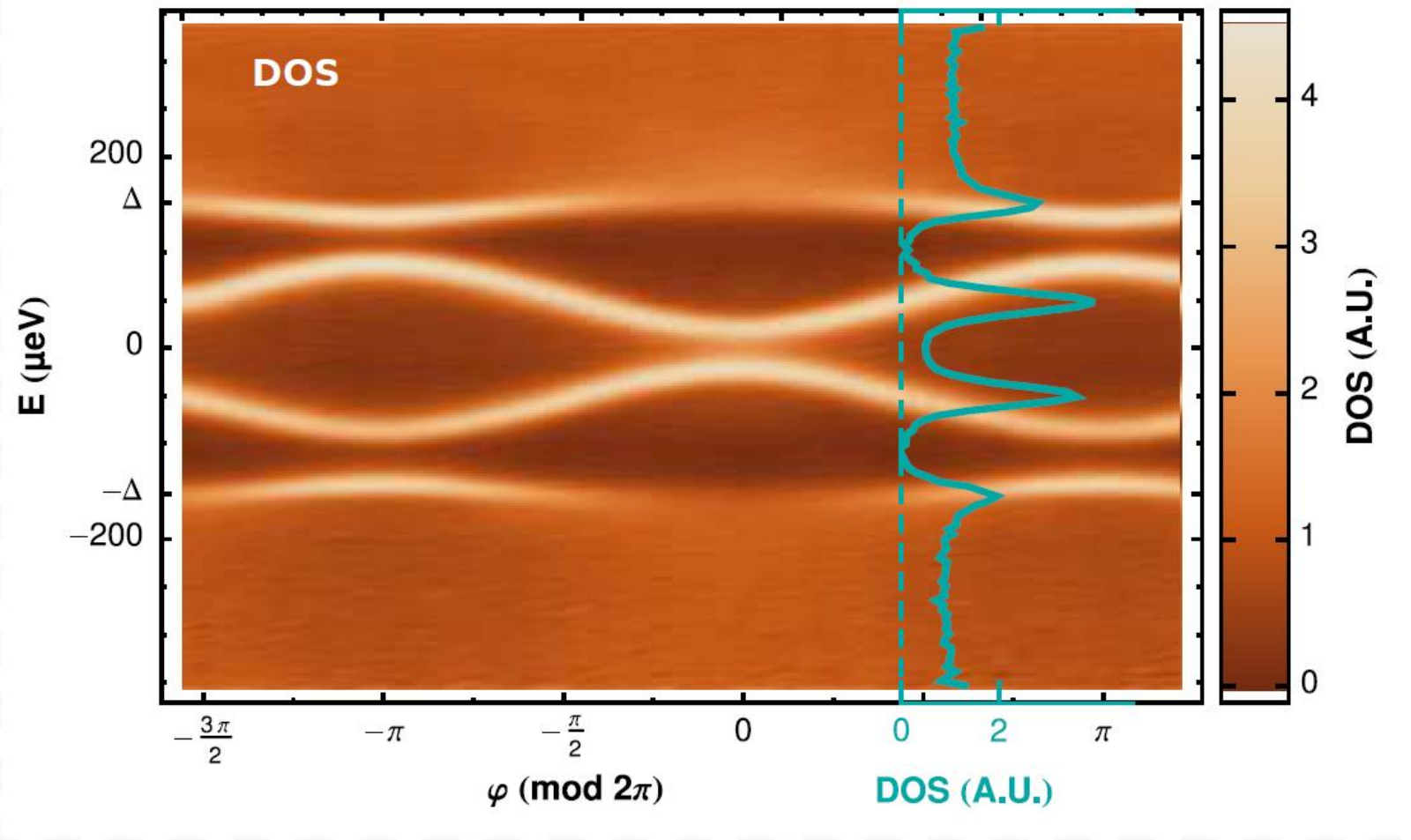
$$g(\varepsilon, V) = \left(f_{NT}(\varepsilon + eV) - f_P(\varepsilon) \right) \rho_P'(\varepsilon) - f_P'(\varepsilon)\, \rho_P(\varepsilon)$$

TUNNELING DENSITY OF STATES (TDOS)



Probe DOS: BCS + small depairing ~ $\Delta/100$

ANDREEV STATES IN CARBON NANOTUBE

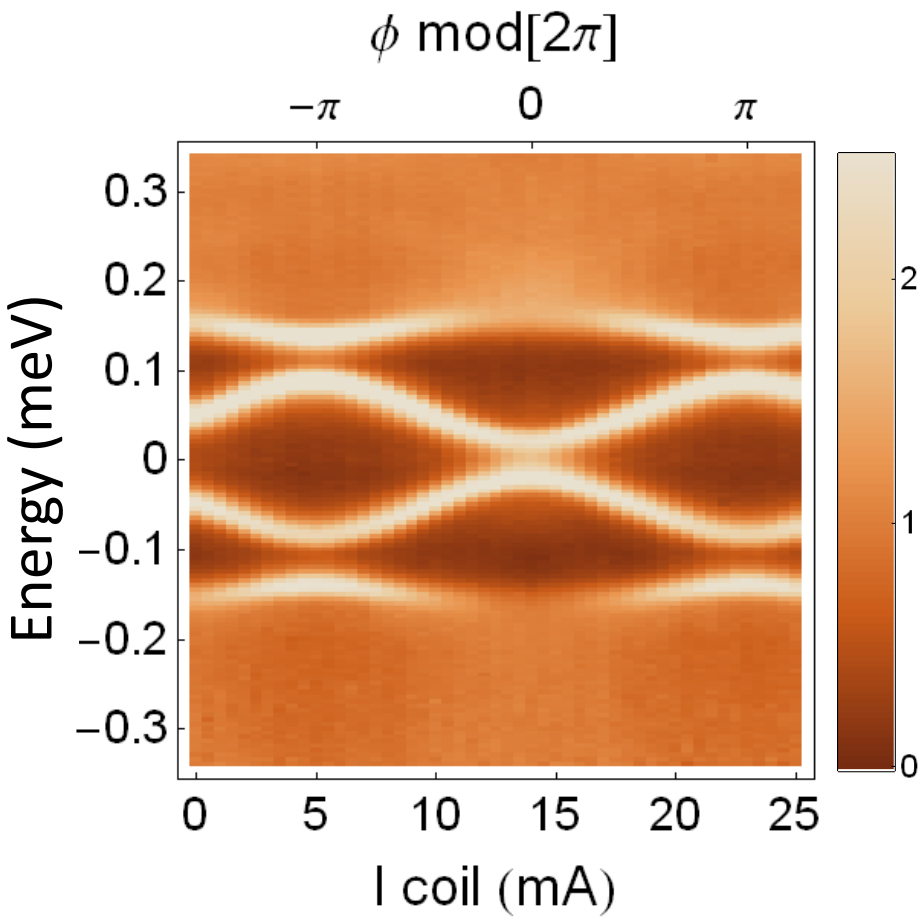


CAVEAT: do not correspond to the short channel model, but are well understood as ABS formed through a quantum dot

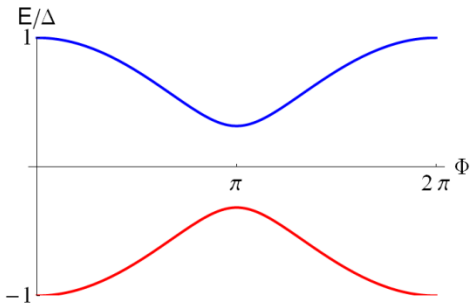
J.-D. Pillet *et al*, PRB **88**, 045101 (2013)

J.-D. Pillet *et al*, Nature Physics (2010)

ABS IN NANOTUBES (IN QDOTS): TDOS FLUX DEPENDENCE



Short single one dimensional channel:

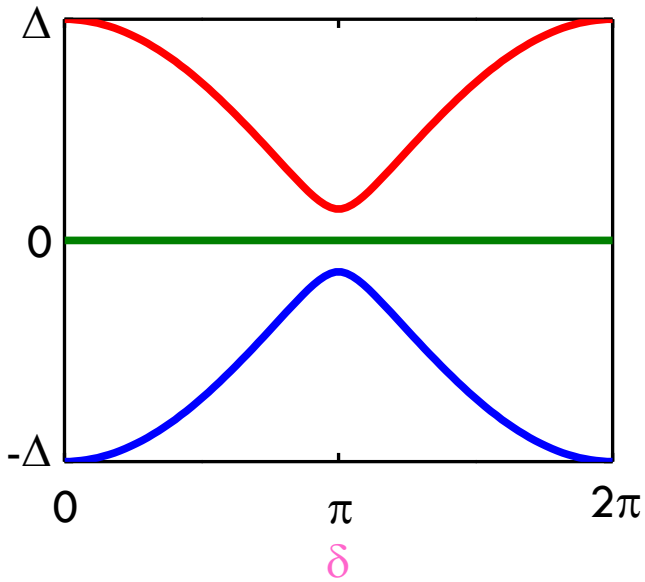


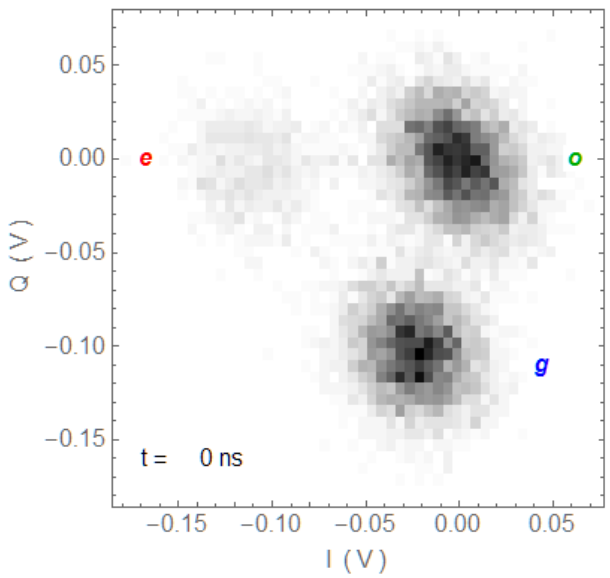
SIMPLE OUTLINE

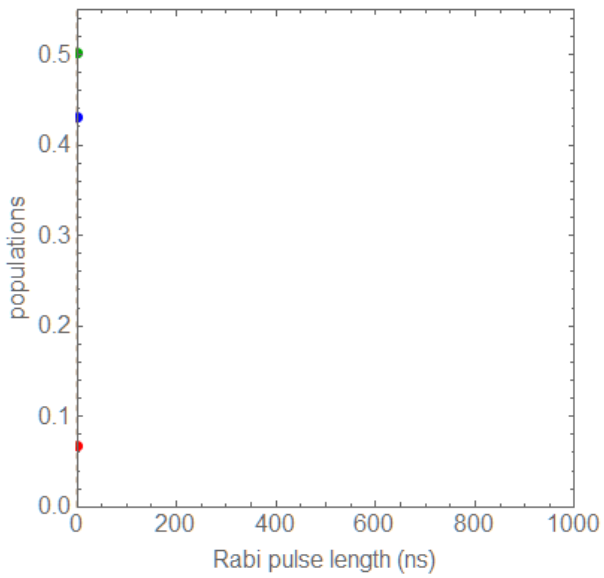
1. ORIGIN OF ABS

2. PROBE ABS

3. USE ABS AS A QUBIT

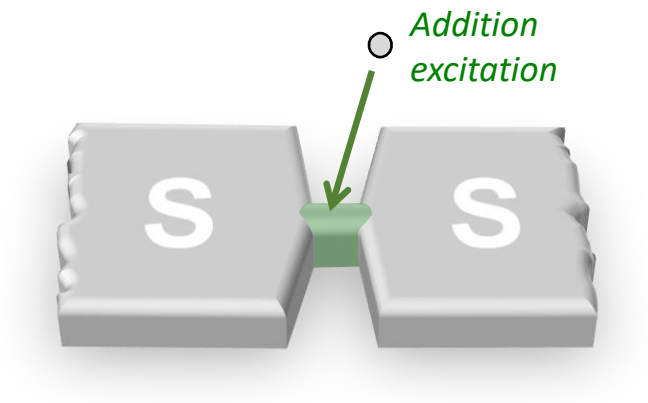




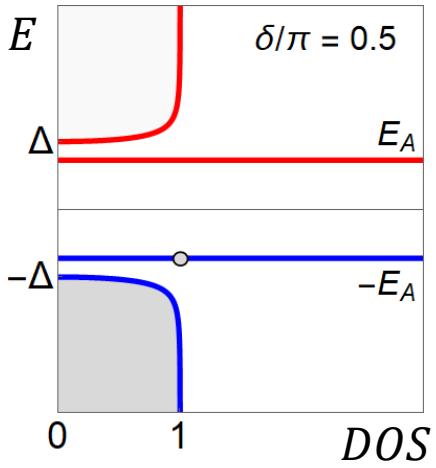


MICROSCOPIC EXCITATIONS IN A SC WEAK-LINK

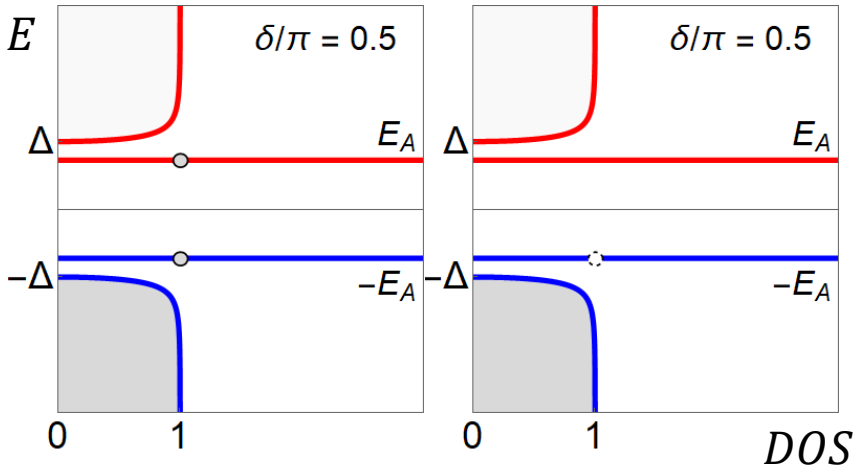
$$\delta \equiv \varphi_L - \varphi_R \neq 0$$



GND State



Excited Odd states

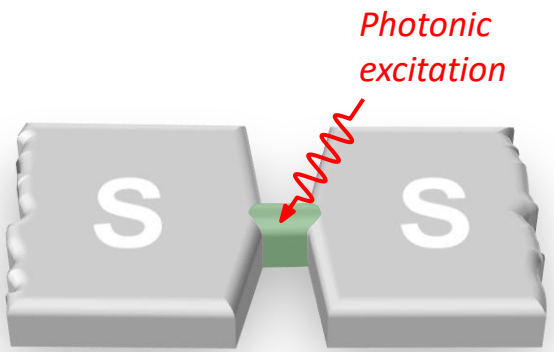


Add a quasiparticle

$$E = E_A$$

MICROSCOPIC EXCITATIONS IN A SC WEAK-LINK

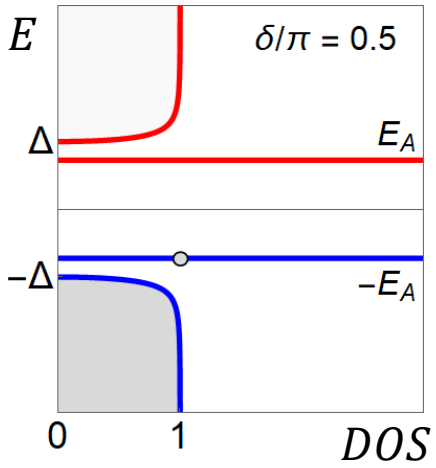
$\delta \equiv \varphi_L - \varphi_R \neq 0$



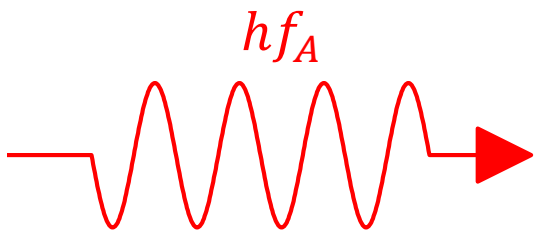
$\Longrightarrow$ **ANDREEV QUBIT**

A. Zazunov, *et al.*Phys. Rev. Lett. (2003).
N. M. Chtchelkatchev and Yu. V. Nazarov, Phys. Rev. Lett. (2003).

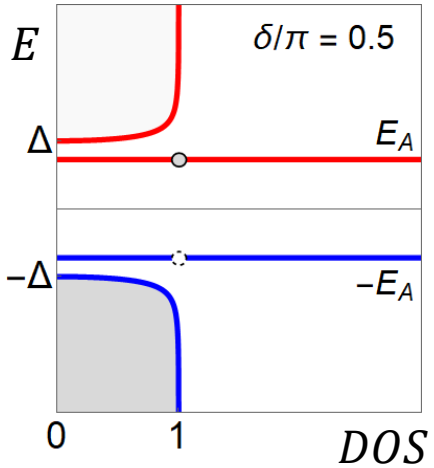
GND State



Excited Odd states



Excited even state

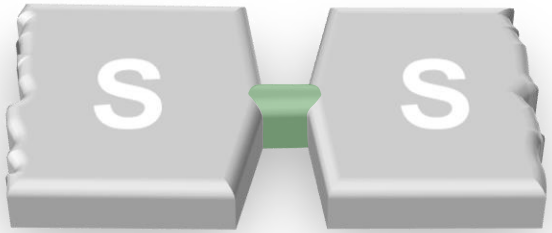


$hf_A = 2E_A$

2 QP excitation

MICROSCOPIC EXCITATIONS IN A SC WEAK-LINK

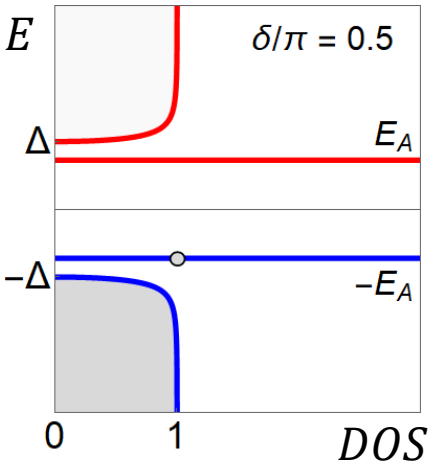
$\delta \equiv \varphi_L - \varphi_R \neq 0$



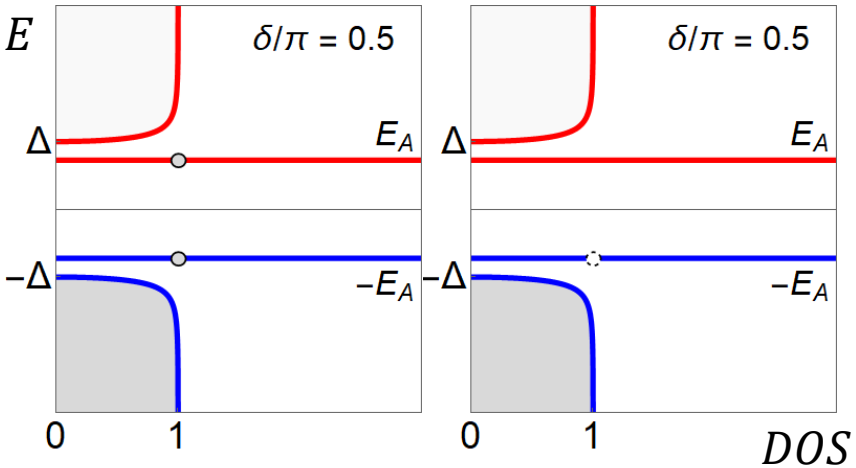
$\Longrightarrow$ **ANDREEV QUANTUM DOT**

A. Zazunov, *et al.*Phys. Rev. Lett. (2003).
N. M. Chtchelkatchev and Yu. V. Nazarov, Phys. Rev. Lett. (2003).

GND State



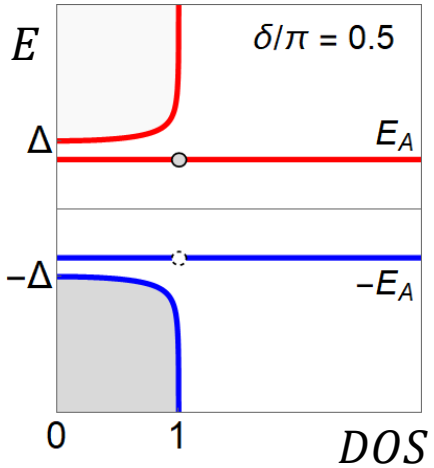
Excited Odd states



$E = E_A$

1 QP excitation

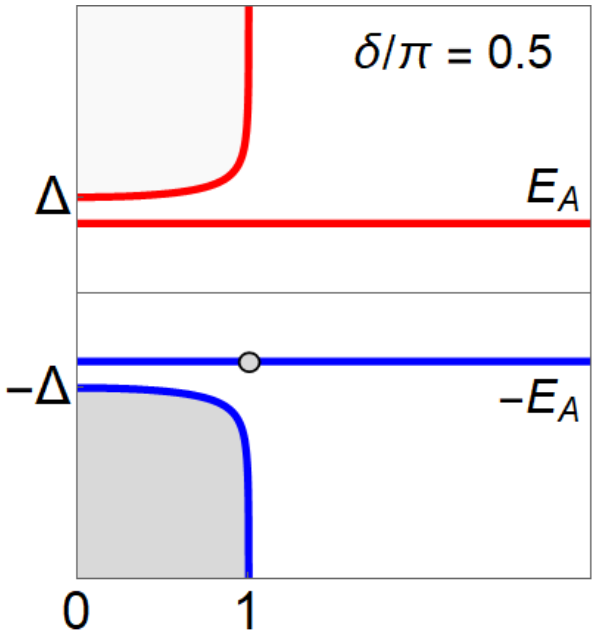
Excited even state



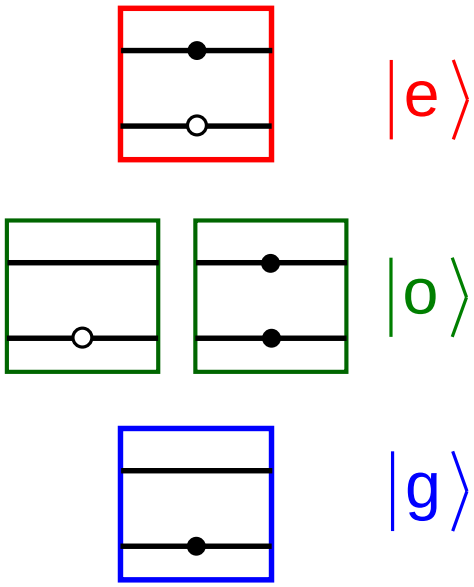
$E = 2E_A$

2 QP excitation

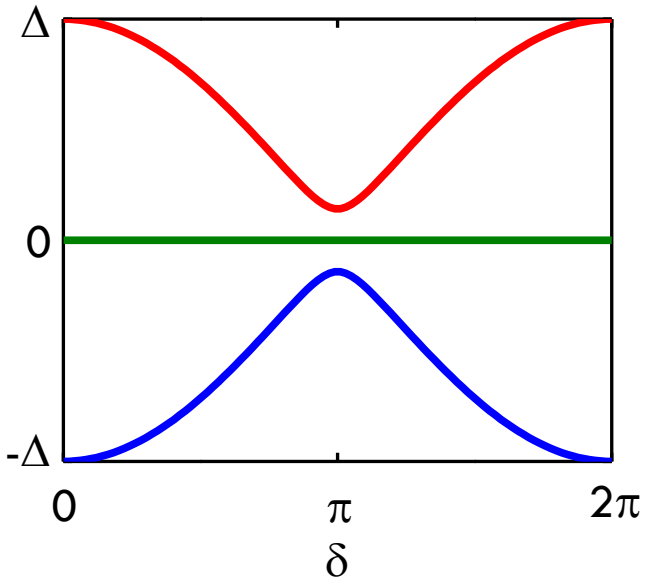
GROUND STATE AND ELEMENTARY EXCITATIONS



LEVELS



OCCUPATION CONFIGURATION



ENERGY OF STATES

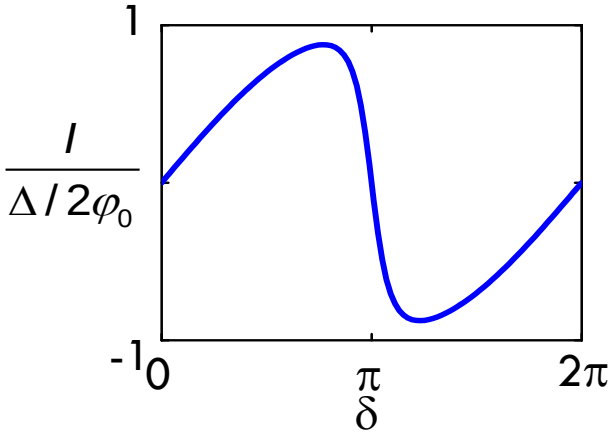
$|e\rangle$ DOUBLE EXCITATION

$|o\rangle$ SINGLE EXCITATION

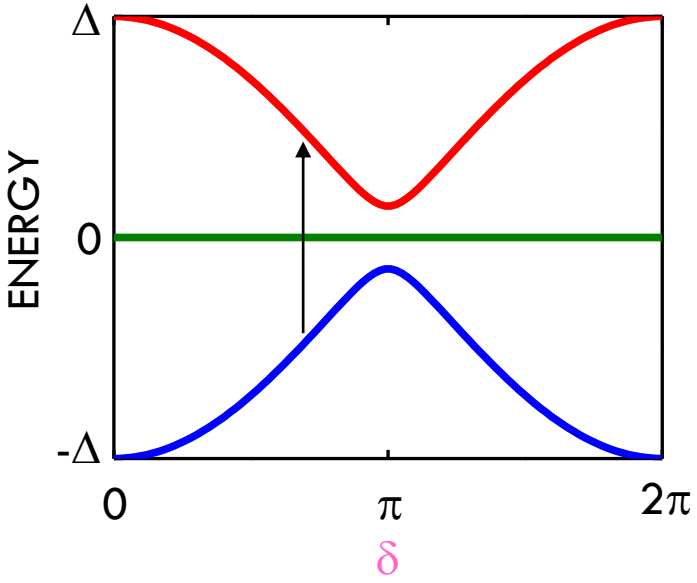
$|g\rangle$ NO EXCITATION

PROBE MESOSCOPIC THEORY OF JOSEPHSON EFFECT

- probe $I(\delta)$ in $|g\rangle$



- Spectroscopy of transitions
- **Drive coherent oscillations between states**

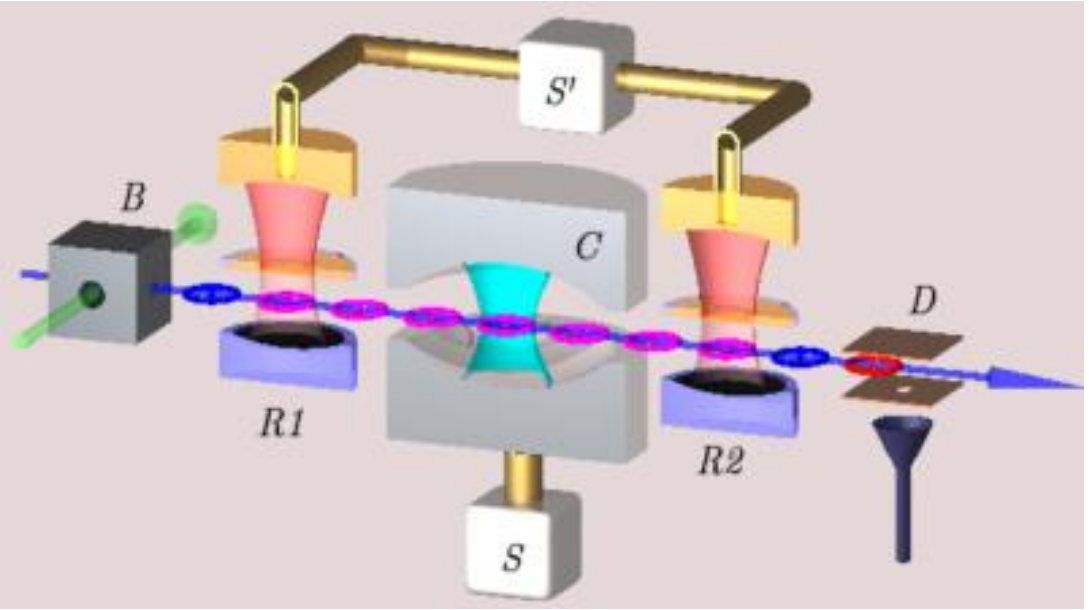


➔ requires:

1. Well-controlled system:
   few channels, measurable transmissions
2. Phase biasing
3. Detection of transitions

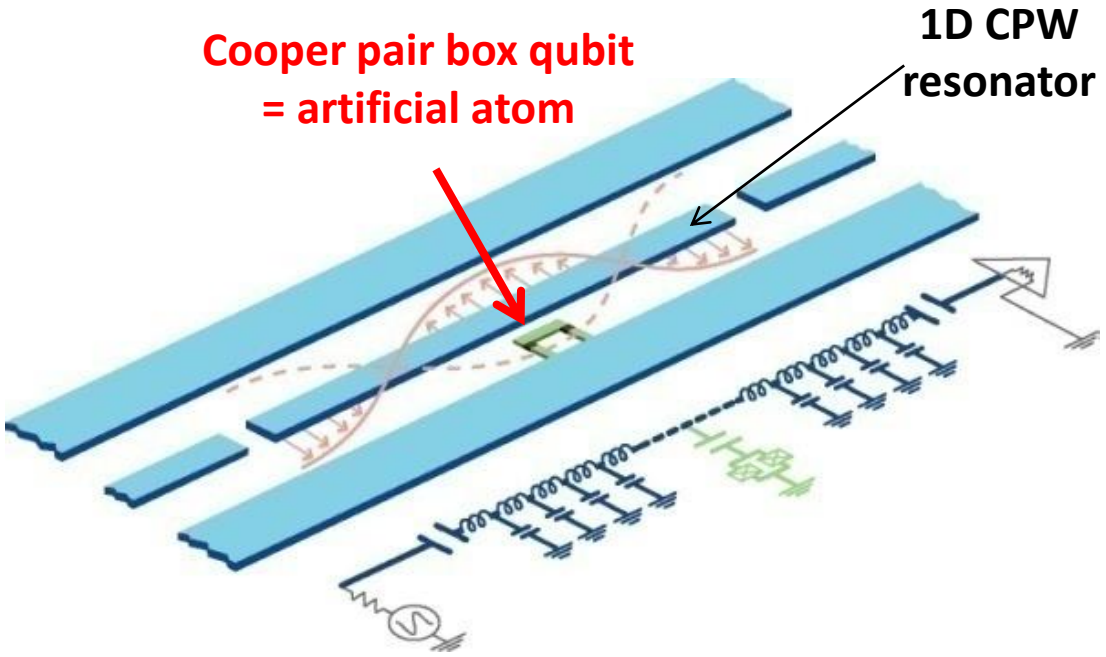
STATE READ-OUT: CIRCUIT QED SETUP

CAVITY QED



©LKB, Brune/Haroche/Raimond

CIRCUIT QED

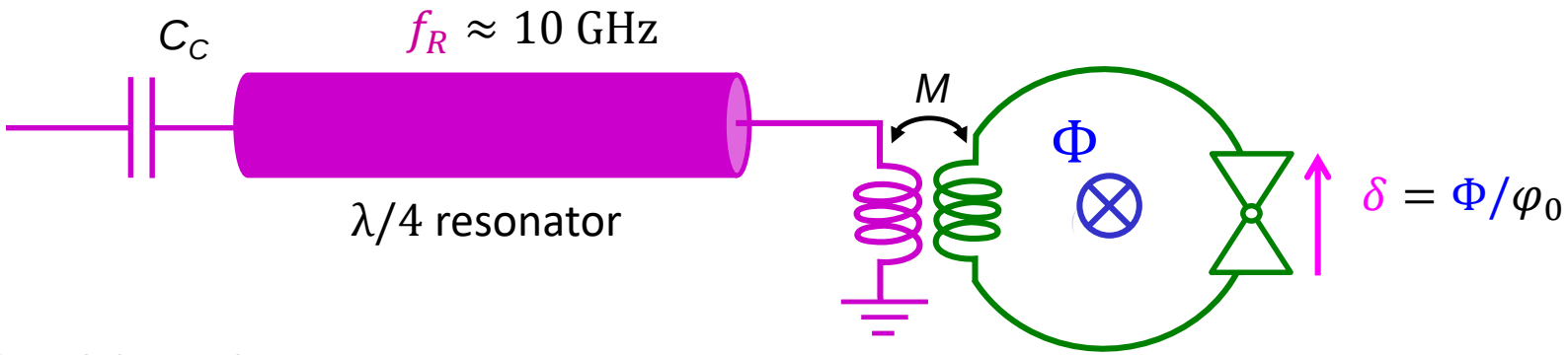


A. Blais *et al.*, Phys. Rev. A (2004)
A. Walraff *et al.*, Nature (2004)
I. Chiorescu *et al.*, Nature (2004)

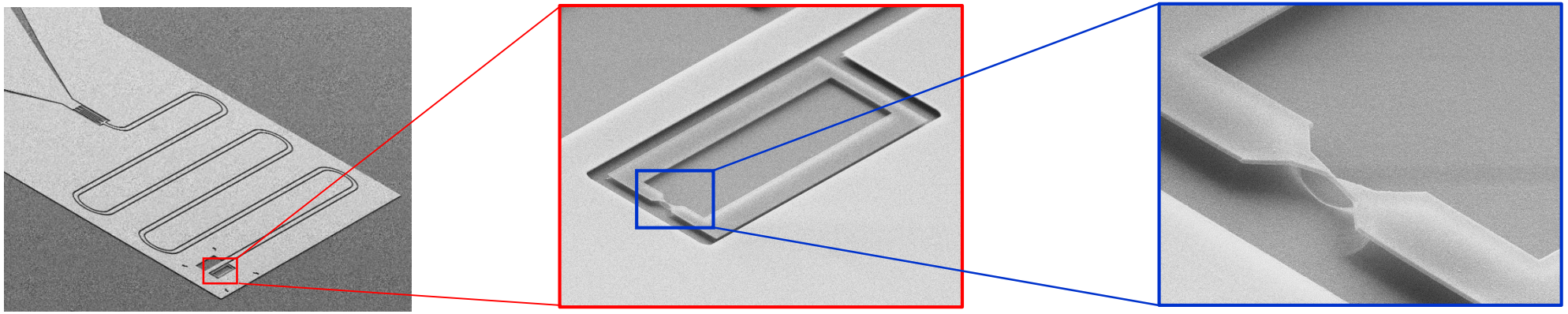
Couple Andreev Quantum Dot to a resonator like atoms in circuit QED

CIRCUIT QED SET-UP

Couple Andreev Quantum Dot to a **resonator** like atoms in cQED

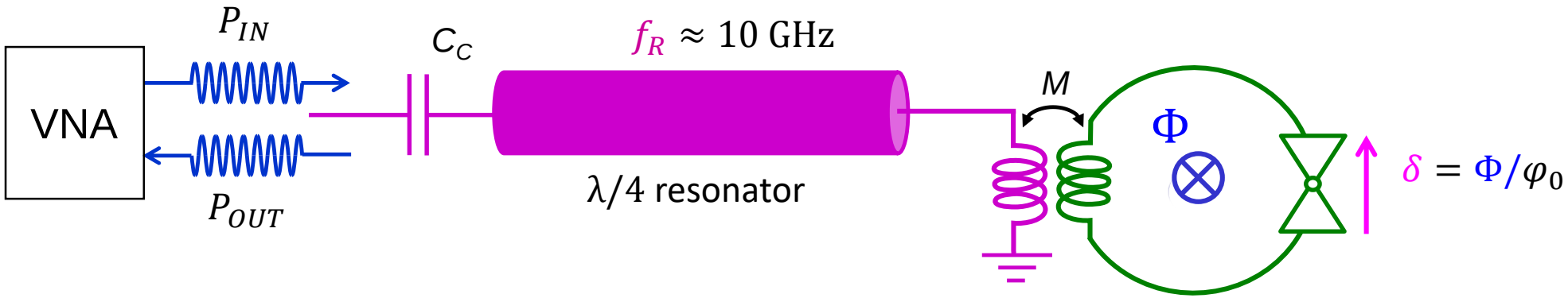


Implementation in a flexible substrate



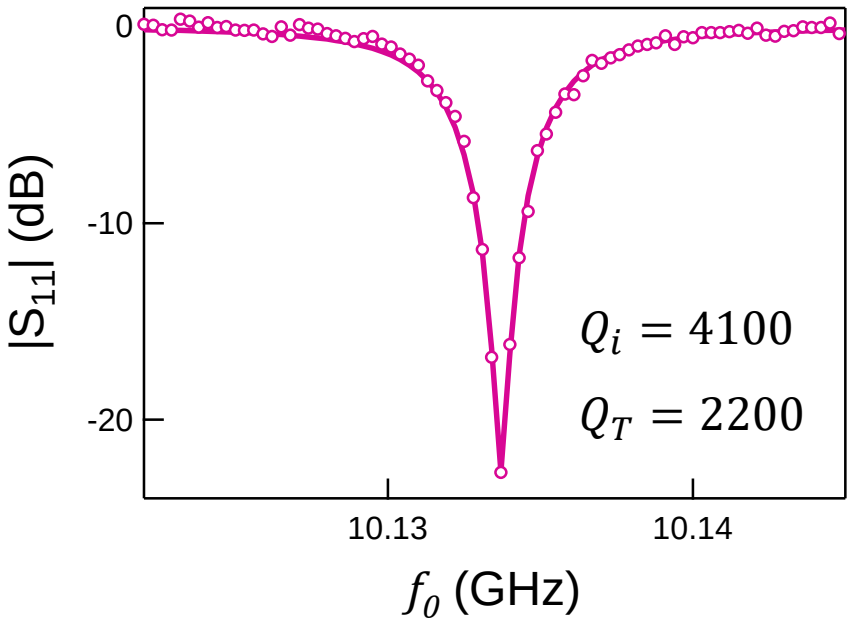
RESONATOR RESPONSE DEPENDS ON THE OCCUPATION OF ABS

RESONATOR CHARACTERIZATION

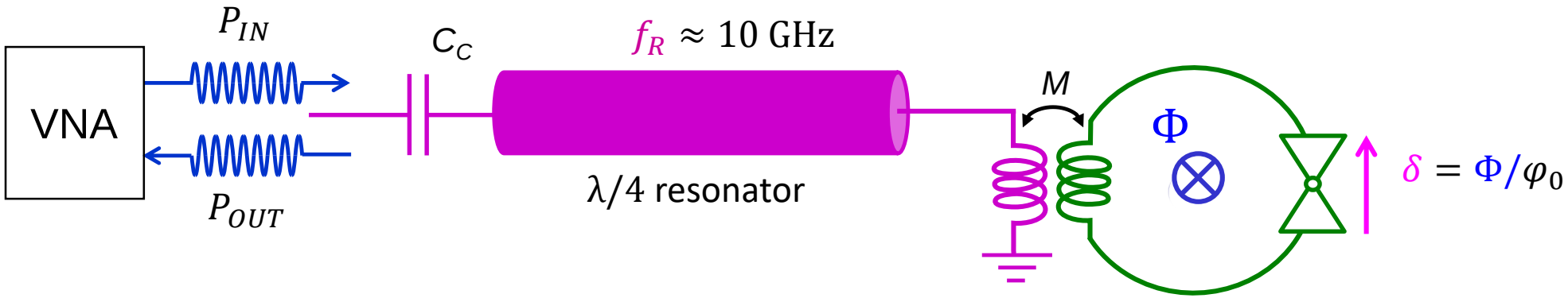


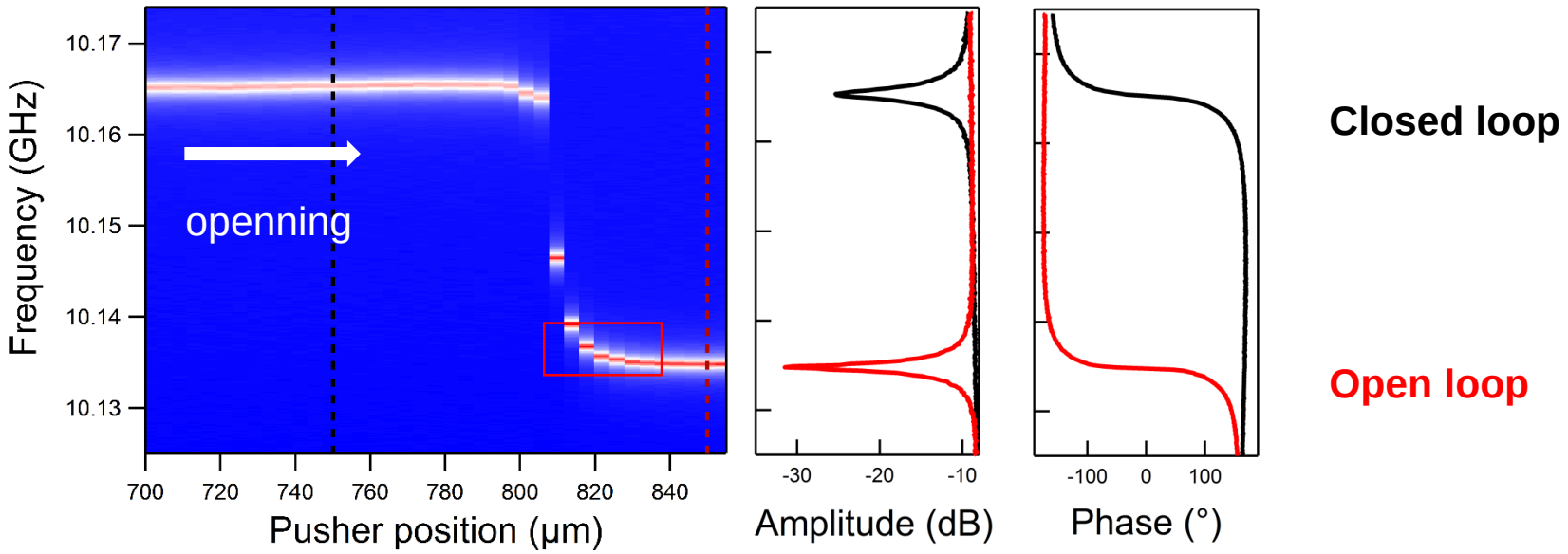
Reflection coefficient

$$S_{11}(f_0) \equiv \frac{P_{OUT}}{P_{IN}}$$



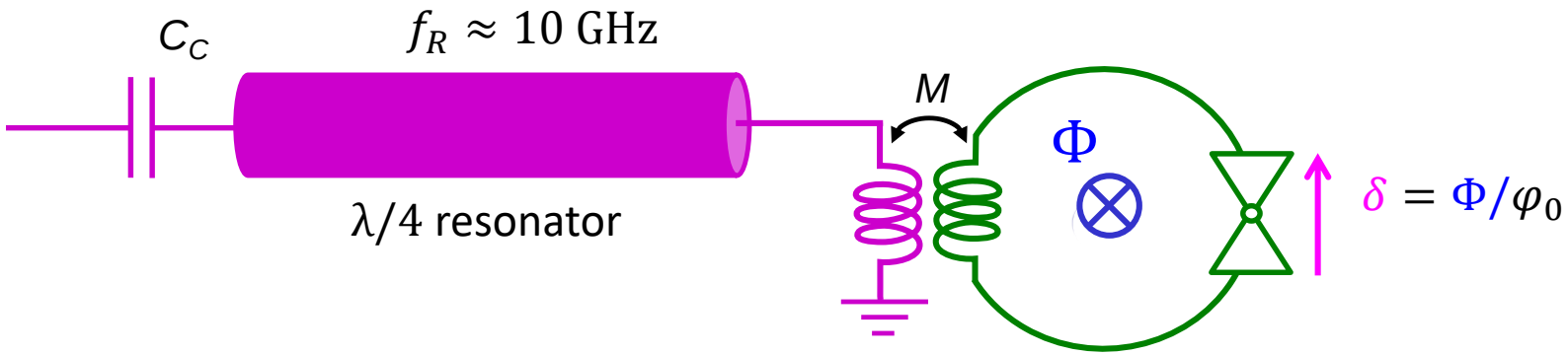
GETTING ONE-ATOM CONTACTS





CONTACTS WITH SMALL NUMBER OF CHANNELS

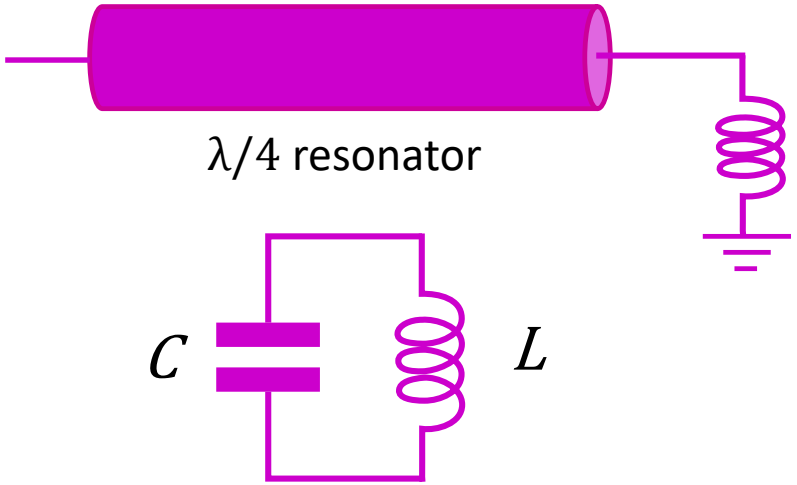
QUANTUM DESCRIPTION OF THE CIRCUIT



$\Rightarrow$ **Hamiltonian of the circuit**

QUANTUM DESCRIPTION OF μ-WAVE RESONATOR

$\lambda/4 \equiv LC$ electrical harmonic oscillator near f_R the resonance frequency



$$f_R = 1/\ 2\pi\sqrt{LC}$$

QUANTUM DESCRIPTION OF μ-WAVE RESONATOR

$[\hat{\phi}, \hat{Q}] = i\hbar$ Lowering & rising operators

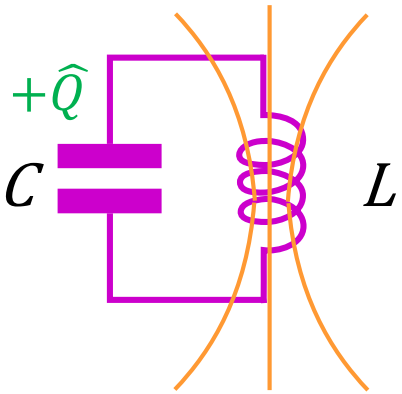
$$\hat{Q} = -ie\sqrt{\frac{R_Q}{\pi Z_R}}\left(a - a^\dagger\right)$$

$$\hat{\phi} = \varphi_0\sqrt{\frac{\pi Z_R}{R_Q}}\left(a + a^\dagger\right)$$

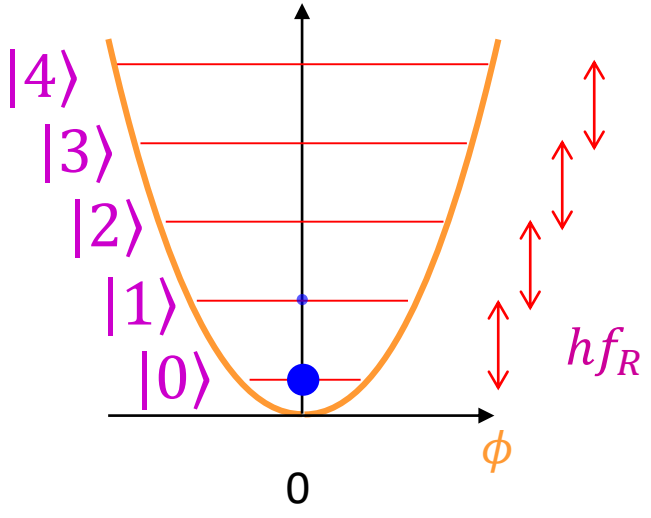
$Z_R = \sqrt{L/C}$; $R_Q = h/(2e)^2$

$f_R = 1/2\pi\sqrt{LC}$

$$\hat{\phi} = \int_{-\infty}^{t} \hat{V}(t')dt'$$



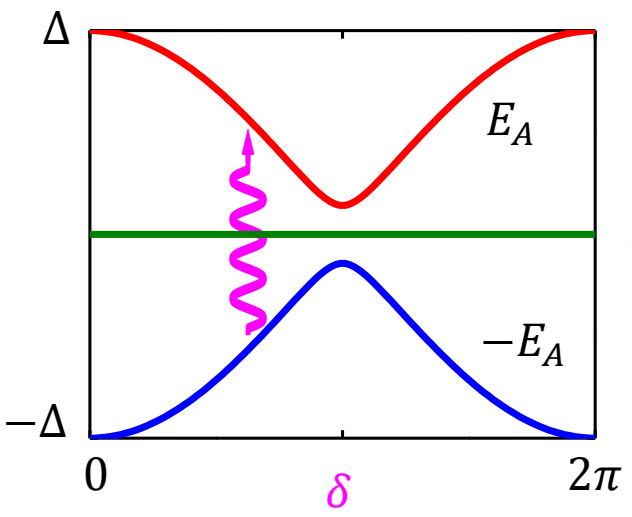
Potential energy $U = \phi^2/2L$



$$\boxed{\mathcal{H} = \frac{1}{2C}\hat{Q}^2 + \frac{1}{2L}\hat{\phi}^2 = hf_R\left(a^\dagger a + 1/2\right)}$$

THE ANDREEV QUBIT

A. Zazunov *et al.*, PRL (2003)

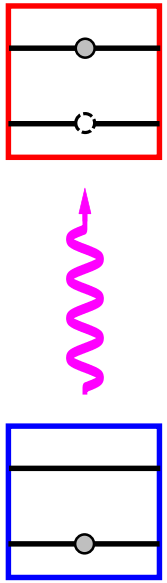


$E_e = +E_A(\delta)$

$E_o = 0$

$E_g = -E_A(\delta)$

$$E_A = \Delta\sqrt{1 - \tau \sin^2(\delta/2)}$$



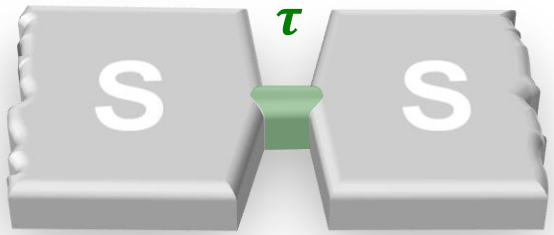
$|e\rangle$ 2 QPs EXCITATION

$hf = hf_A \equiv 2E_A$

$|g\rangle$ NO EXCITATION

OCCUPATION CONFIGURATION

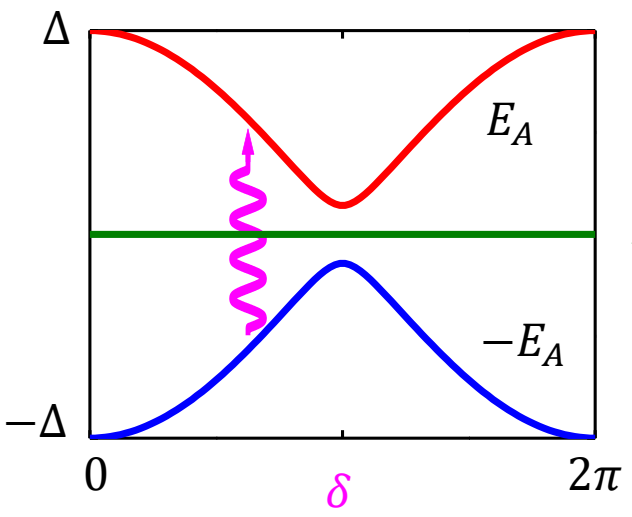
A TWO LEVEL SYSTEM AS LONG AS PARITY IS CONSERVED



$\delta \equiv \varphi_L - \varphi_R \neq 0$

THE ANDREEV QUBIT

A. Zazunov *et al.*, PRL (2003)

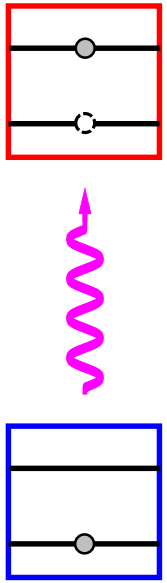


$E_e = +E_A(\delta)$

$E_o = 0$

$E_g = -E_A(\delta)$

$$E_A = \Delta\sqrt{1 - \tau \sin^2(\delta/2)}$$



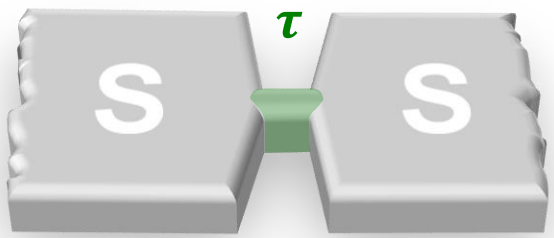
$|e\rangle$ 2 QPs EXCITATION

$hf = hf_A \equiv 2E_A$

$|g\rangle$ NO EXCITATION

OCCUPATION CONFIGURATION

A TWO LEVEL SYSTEM AS LONG AS PARITY IS CONSERVED



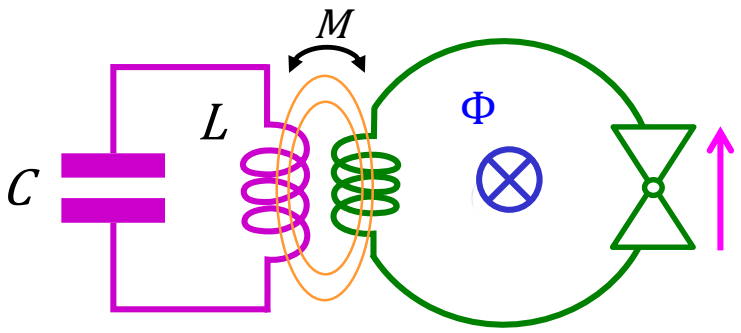
$\delta \equiv \varphi_L - \varphi_R \neq 0$

HAMILTONIAN IN THE $|g\rangle$,$|e\rangle$ BASE

$$\mathcal{H}_A = -\frac{hf_A}{2}(\delta, \tau) \cdot \sigma_z \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

QUANTUM DESCRIPTION OF COUPLED SYSTEM

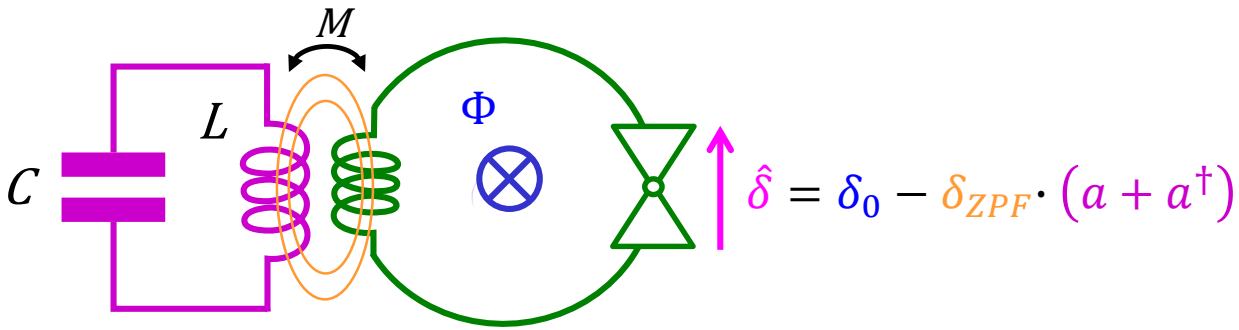
Inductive coupling to phased biased AQD



Phase δ ⇒ quantum operator

$$\hat{\delta} = \delta_0 - \delta_{ZPF} \cdot \left(a + a^{\dagger}\right)$$

QUANTUM DESCRIPTION OF COUPLED SYSTEM



Andreev quantum dot coupled to a resonator: Rabi Hamiltonian

$$\mathcal{H} = hf_R\left(a^\dagger a + 1/2\right) - \frac{1}{2} hf_A \cdot \sigma_z + hg\left(a^\dagger + a\right) \cdot (\sigma^+ + \sigma^-)$$

When $|f_A - f_R| \ll f_A + f_R$; f_R $\Rightarrow$ Jaynes-Cummings Hamiltonian

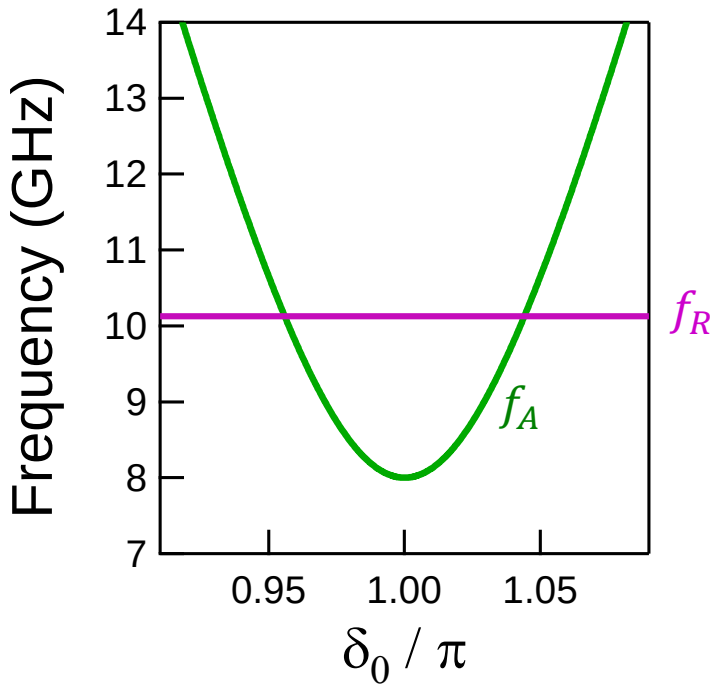
$$\boxed{\mathcal{H} = hf_R\left(a^\dagger a + 1/2\right) - \frac{hf_A}{2} \cdot \sigma_z + hg\left(a^\dagger \sigma^- + a\sigma^+\right)}$$

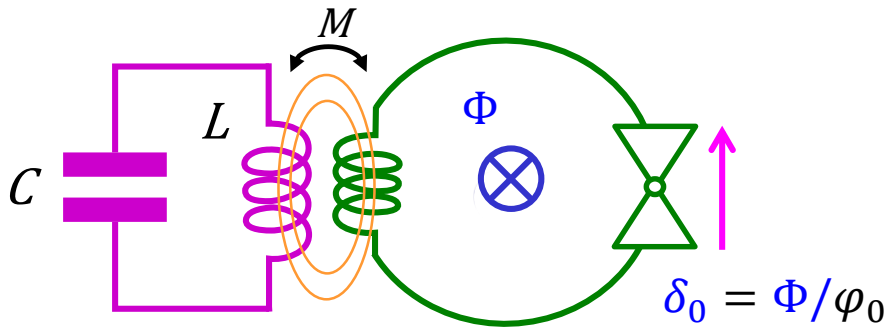
Coupling term
Transfer of a single excitation

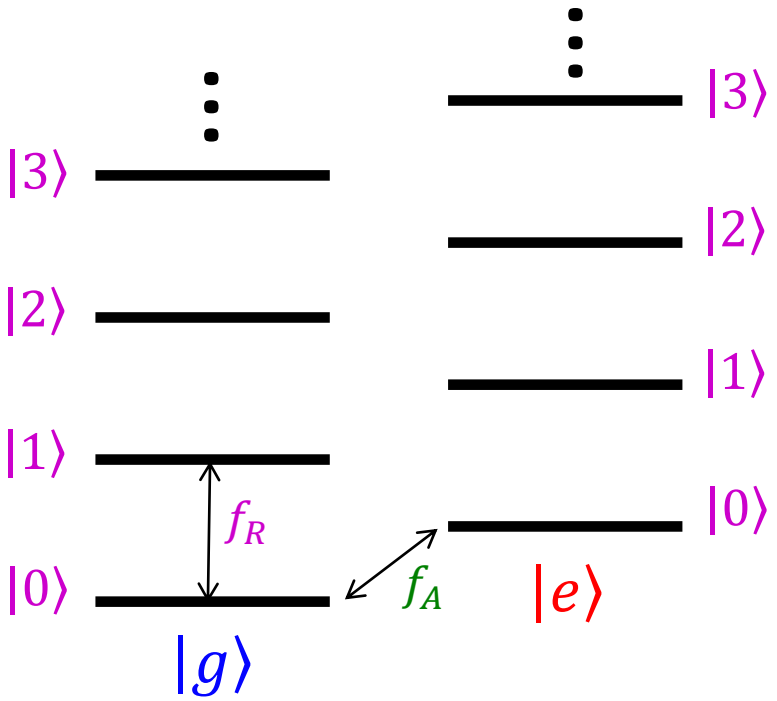
AQD→ Resonator

Resonator →AQD

QUANTUM DESCRIPTION OF COUPLED SYSTEM



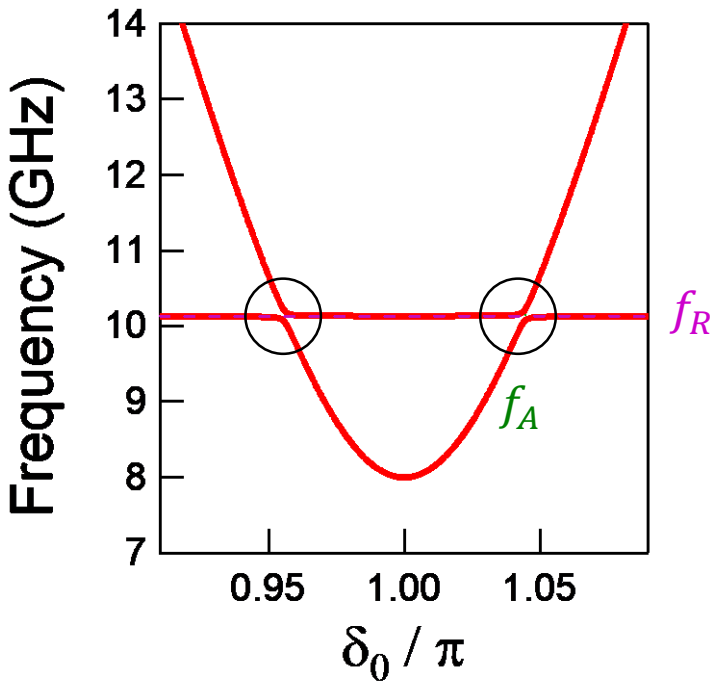


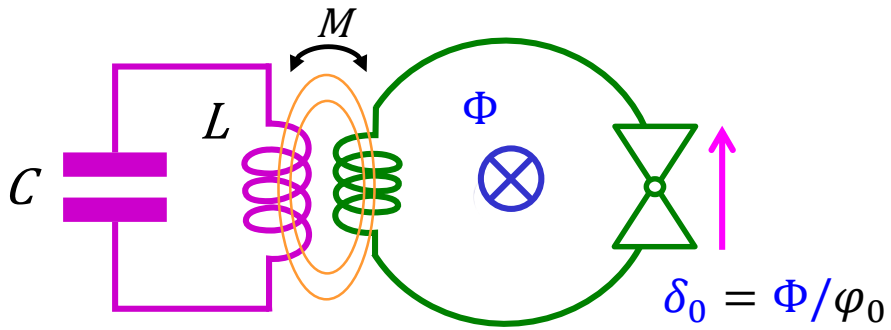


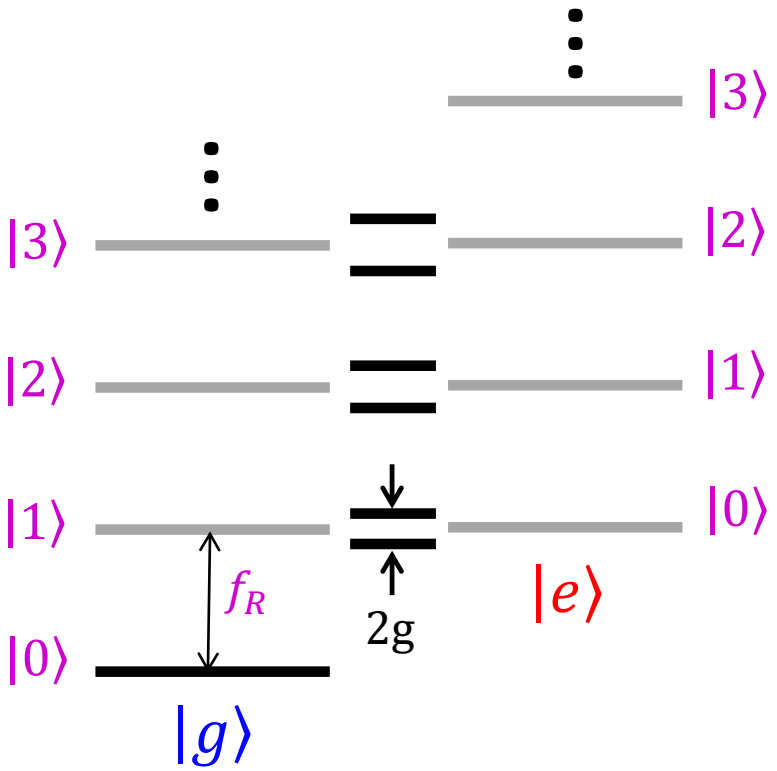
Jaynes-Cummings Hamiltonian

$$\mathcal{H} = hf_R\left(a^\dagger a + 1/2\right) - \frac{hf_A}{2} \cdot \sigma_z + hg\left(a^\dagger \sigma^- + a\sigma^+\right)$$

HYBRIDIZATION AT DEGENERACY POINT







Jaynes-Cummings Hamiltonian

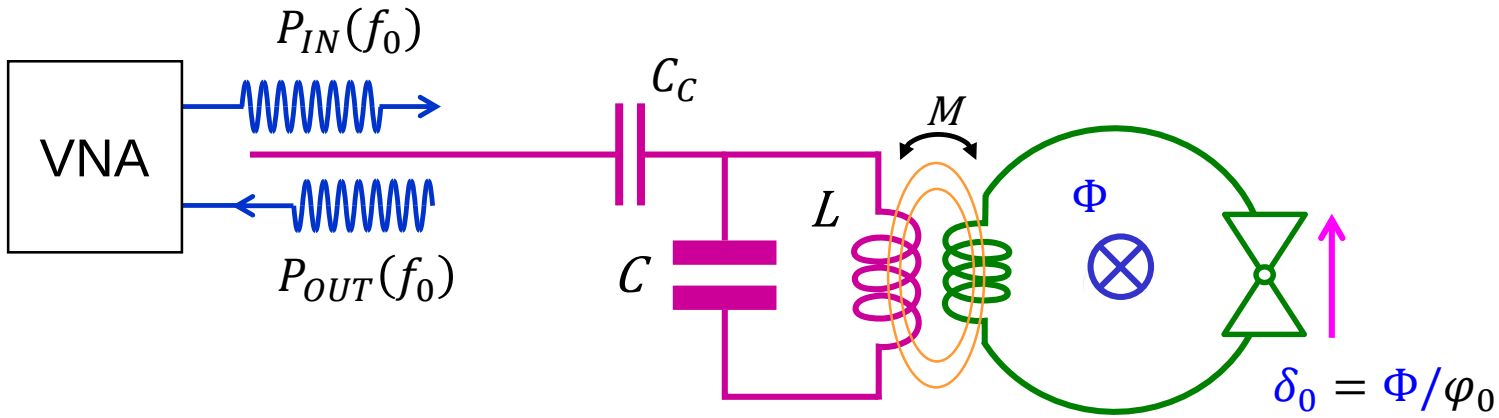
$$\mathcal{H} = hf_R\left(a^\dagger a + 1/2\right) - \frac{hf_A}{2} \cdot \sigma_z + hg\left(a^\dagger \sigma^- + a\sigma^+\right)$$

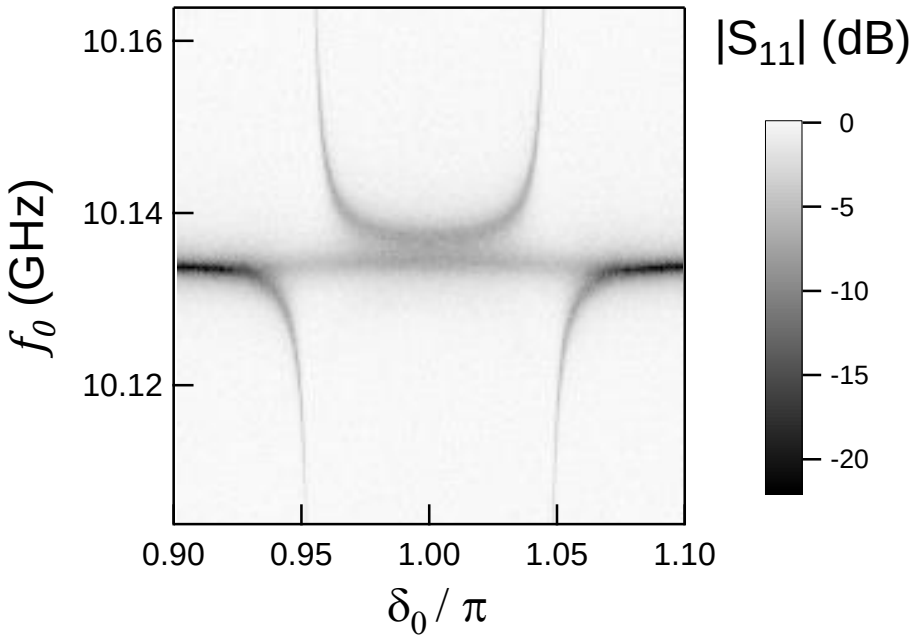
ANTI-CROSSING

Reflection coefficient

$$S_{11}(f_0) \equiv \frac{P_{OUT}}{P_{IN}}$$

$$|P_{IN}| \rightarrow \langle n \rangle < 1$$



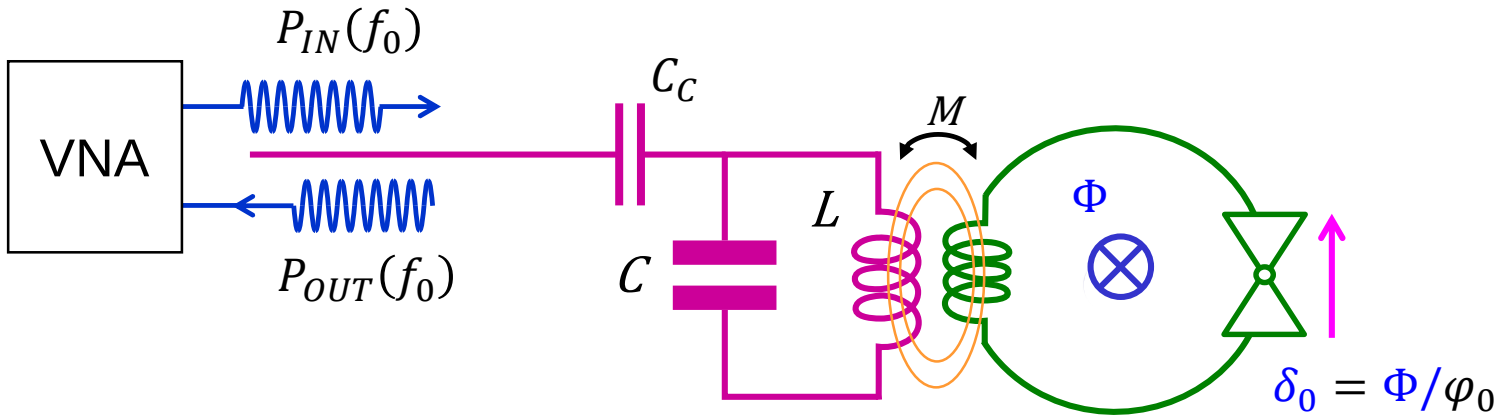


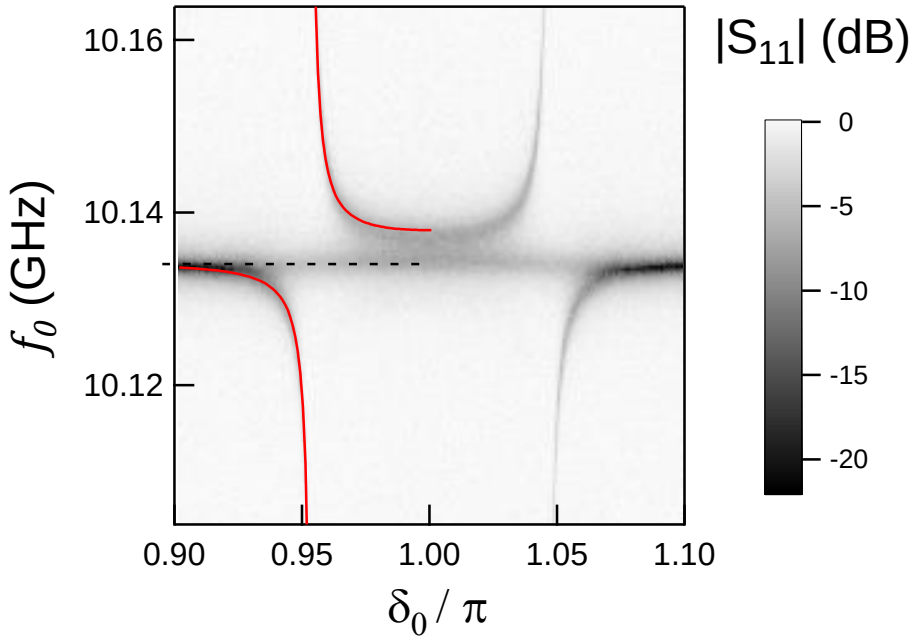
FIT ANTI-CROSSING

Reflection coefficient

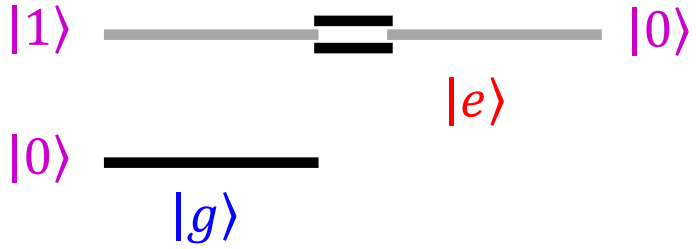
$$S_{11}(f_0) \equiv \frac{P_{OUT}}{P_{IN}}$$

$$|P_{IN}| \rightarrow \langle n \rangle < 1$$





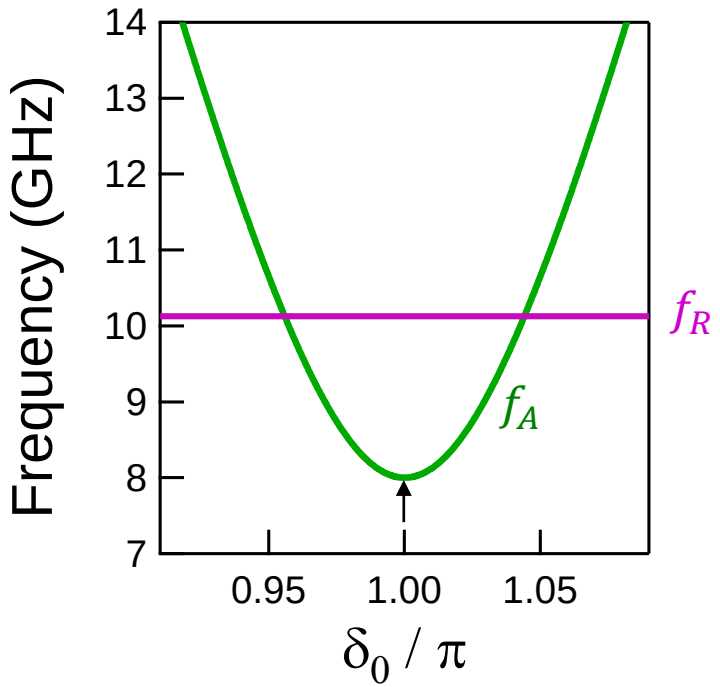
$$\frac{1}{\sqrt{2}}\left(|g,1\rangle \pm |e,0\rangle\right)$$

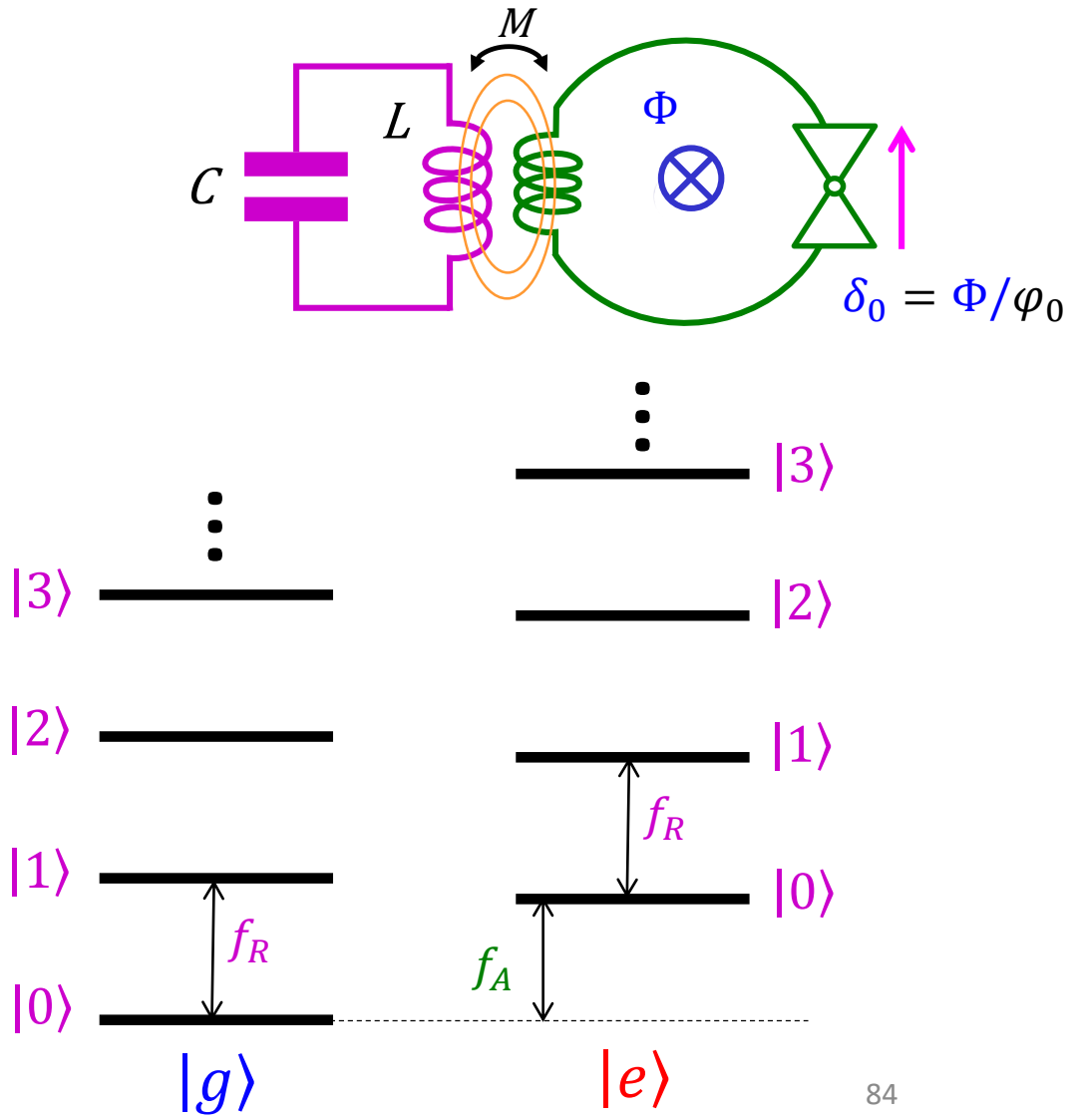


$$\frac{1}{2}\left((f_A + f_R) \pm \sqrt{(f_A - f_R)^2 + 4g^2}\right)$$

$g \cong 74\ MHz \qquad \tau \cong 0.9922$

STATE READ-OUT IN DISPERSIVE LIMIT

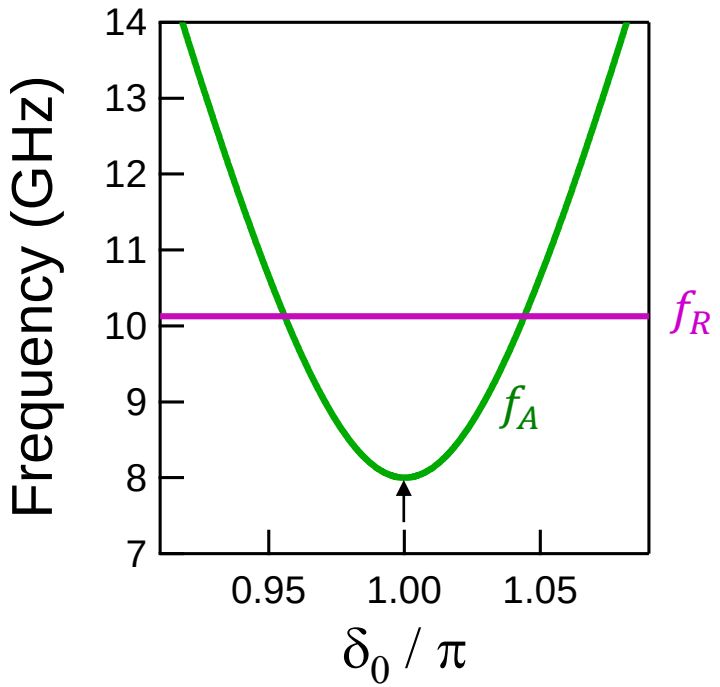


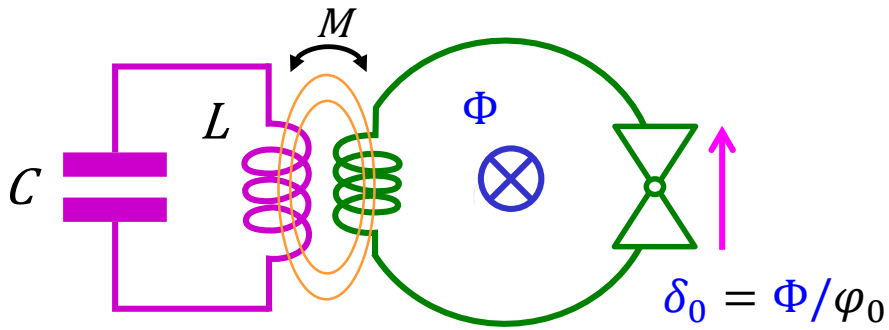


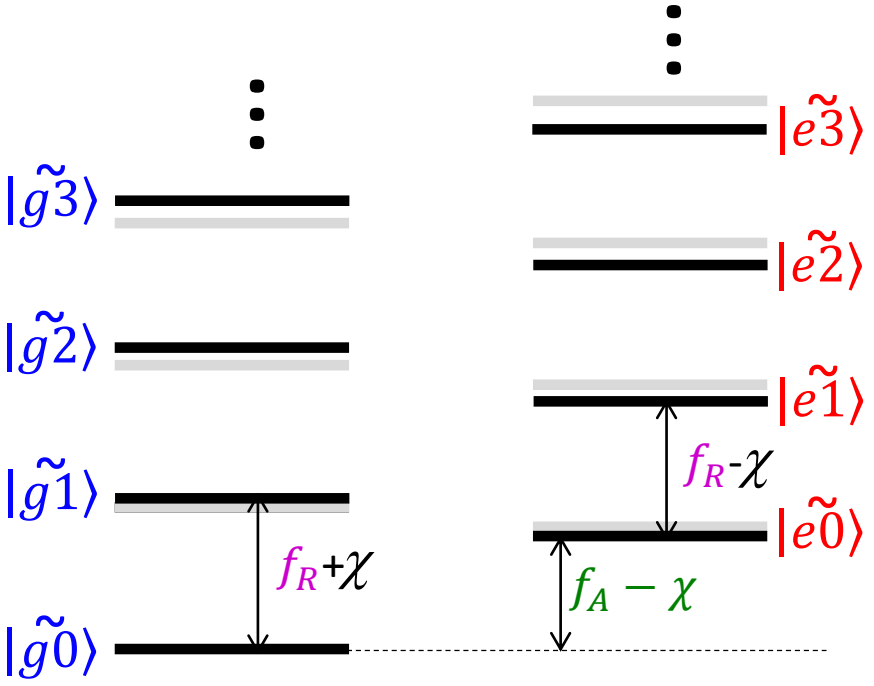
Jaynes-Cummings Hamiltonian

$$\mathcal{H} = hf_R\left(a^\dagger a + 1/2\right) - \frac{hf_A}{2} \cdot \sigma_z + hg\left(a^\dagger \sigma^- + a\sigma^+\right)$$

DRESSED STATES







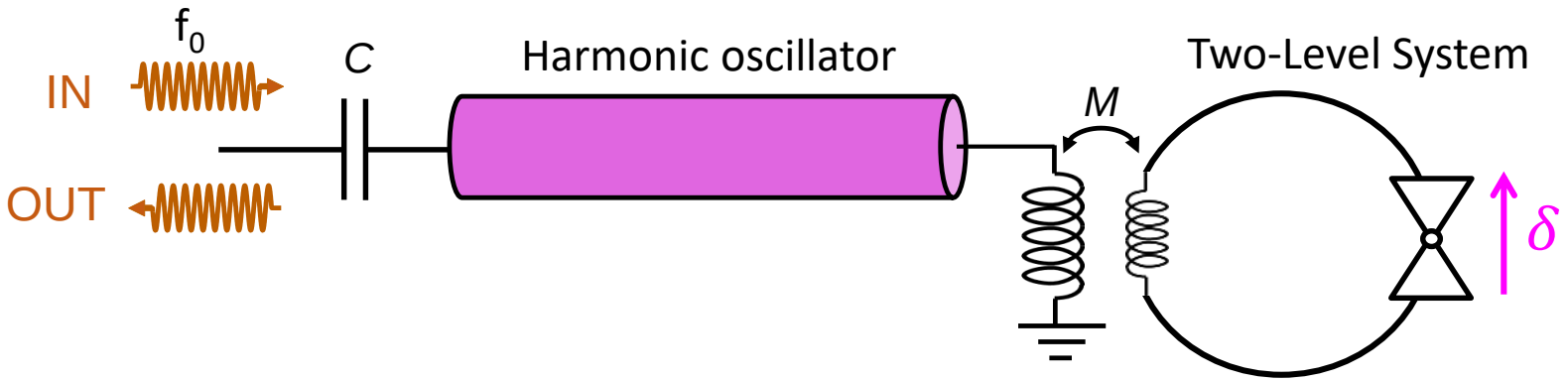
Jaynes-Cummings Hamiltonian in dispersive limit

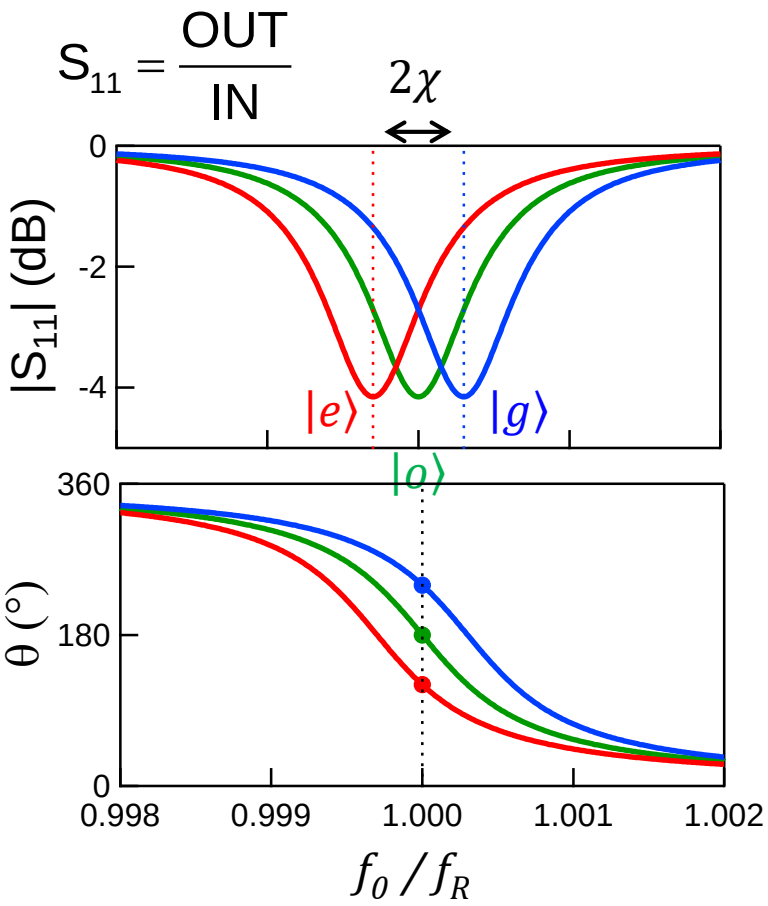
$$\mathcal{H}_{disp} = h(f_R - \chi \cdot \sigma_z)\, a^\dagger a - \frac{h}{2}(f_A - \chi) \cdot \sigma_z$$

$$\chi \equiv \frac{g^2}{f_A - f_R}$$

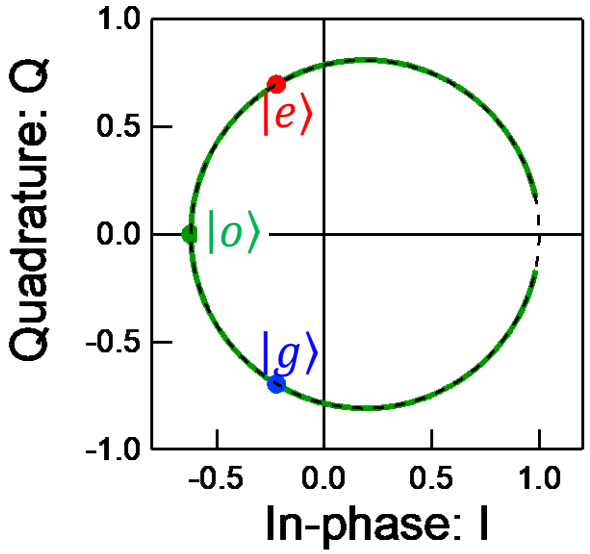
Frequency of resonator depends on state (e/g): *dispersive shift*

DISPERSIVE READ-OUT



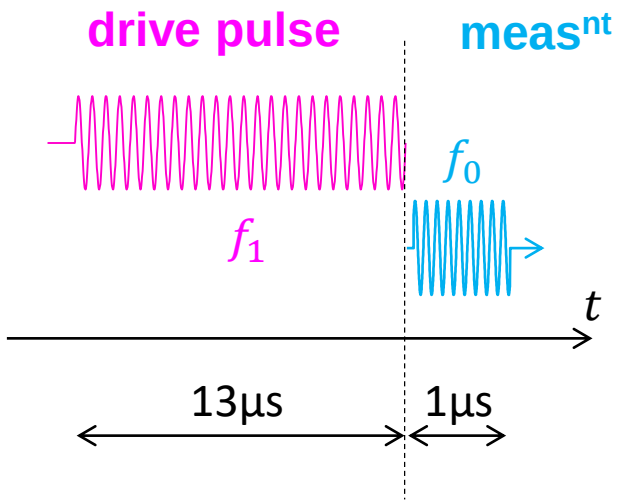


IQ-PLANE REPRESENTATION



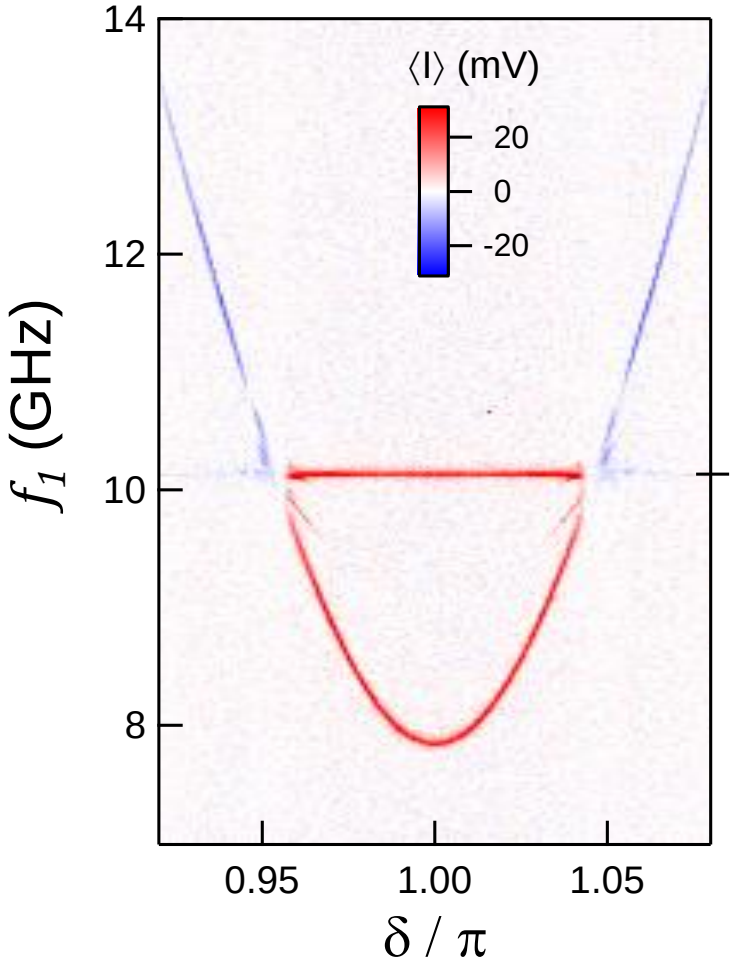
TWO-TONE SPECTROSCOPY

DETECT CHANGES OF THE RESONATOR FREQUENCY WHEN EXCITING THE ANDREEV TRANSITION



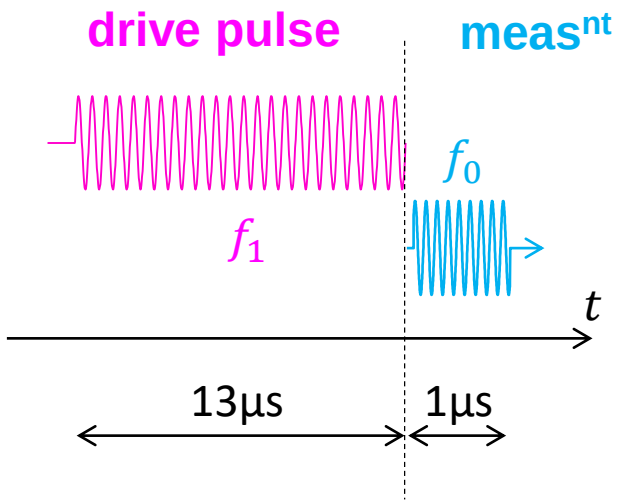
READ-OUT $Q(t)$; $I(t)$

REPEAT, AVERAGE



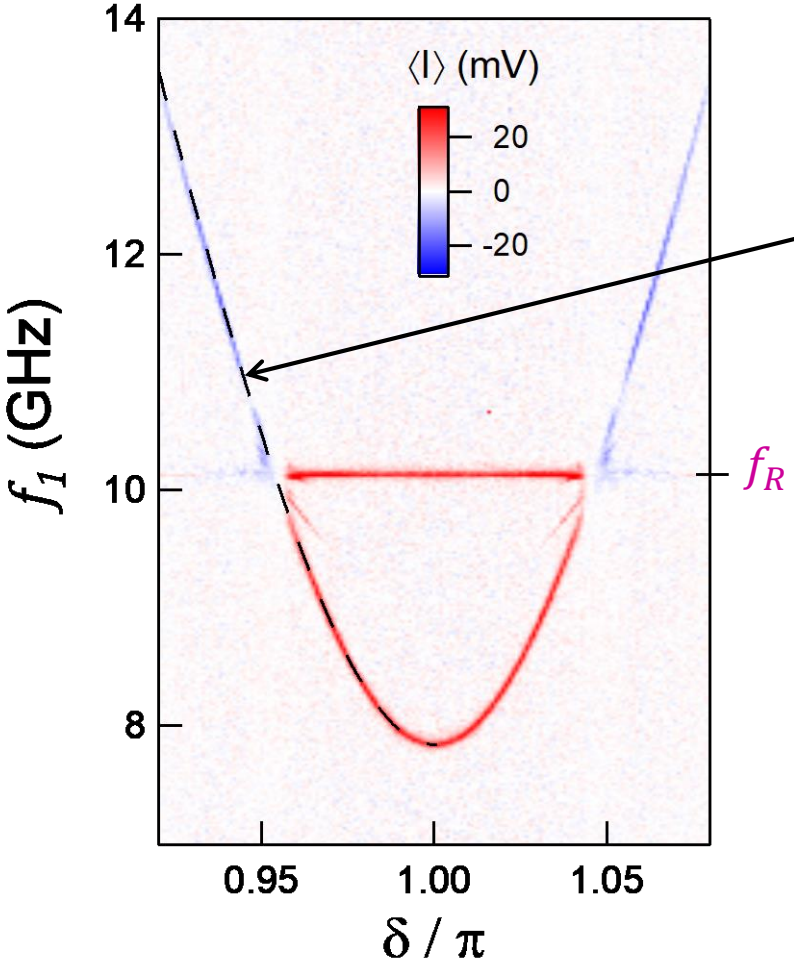
C. Janvier *et al.*, Science **349**, 1199 (2015)

SPECTROSCOPY OF ANDREEV TRANSITION



READ-OUT $Q(t)$; $I(t)$

REPEAT, AVERAGE



$$hf_A = 2\Delta\sqrt{1-\tau\cdot\sin^2(\delta/2)}$$

$\tau = 0.99217$

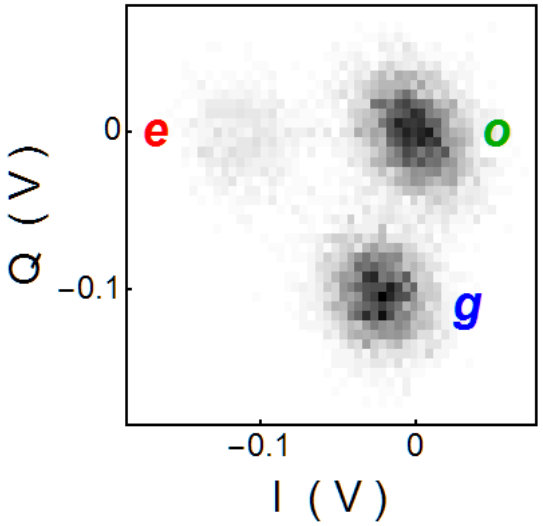
$\Delta = 44.3$ GHz

SINGLE-SHOT DETERMINATION OF STATE

$\delta = \pi$

DENSITY PLOT
FOR ~10^4 PULSES

$\langle n \rangle \cong 30$



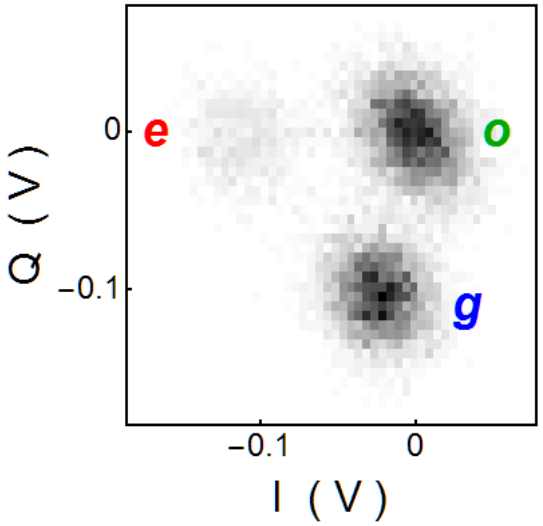
NUMBER OF POINTS IN EACH CLOUD
GIVES POPULATION OF EACH STATE

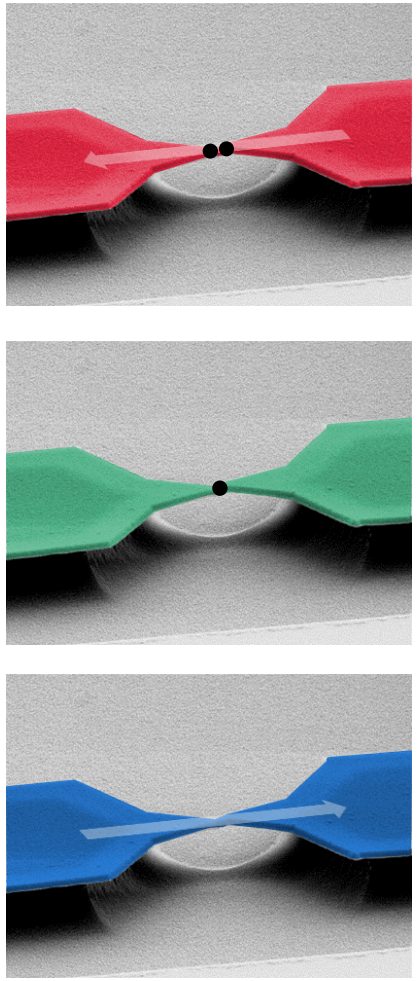
SINGLE-SHOT DETERMINATION OF STATE

$\delta = \pi$

$\langle n \rangle \cong 30$

DENSITY PLOT
FOR ~10^4 PULSES

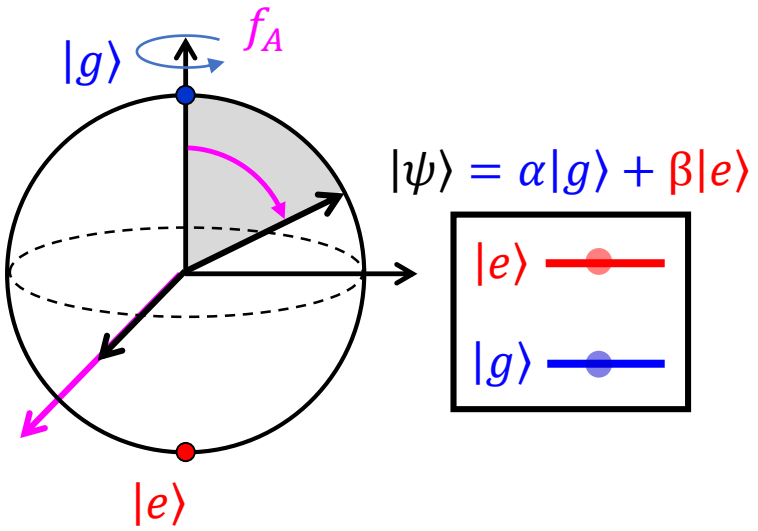




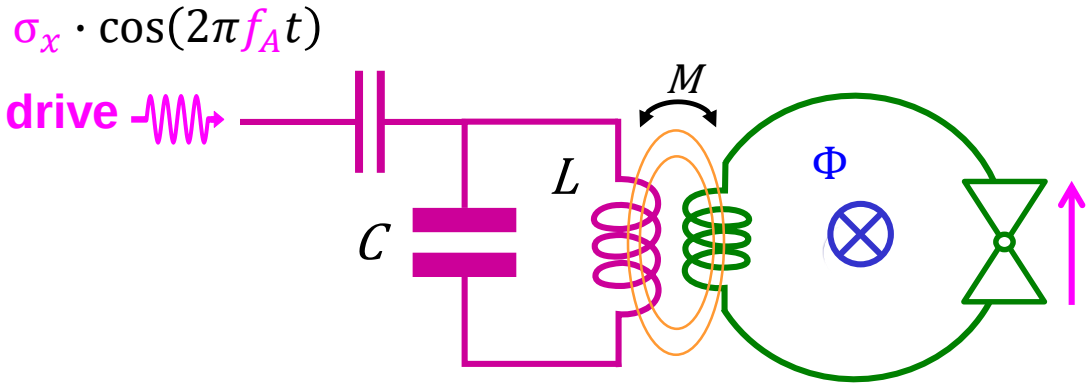
NUMBER OF POINTS IN EACH CLOUD
GIVES POPULATION OF EACH STATE

COHERENT MANIPULATION AT δ=π

Hilbert space of AQD two-level system ($|g\rangle$, $|e\rangle$) → Bloch sphere



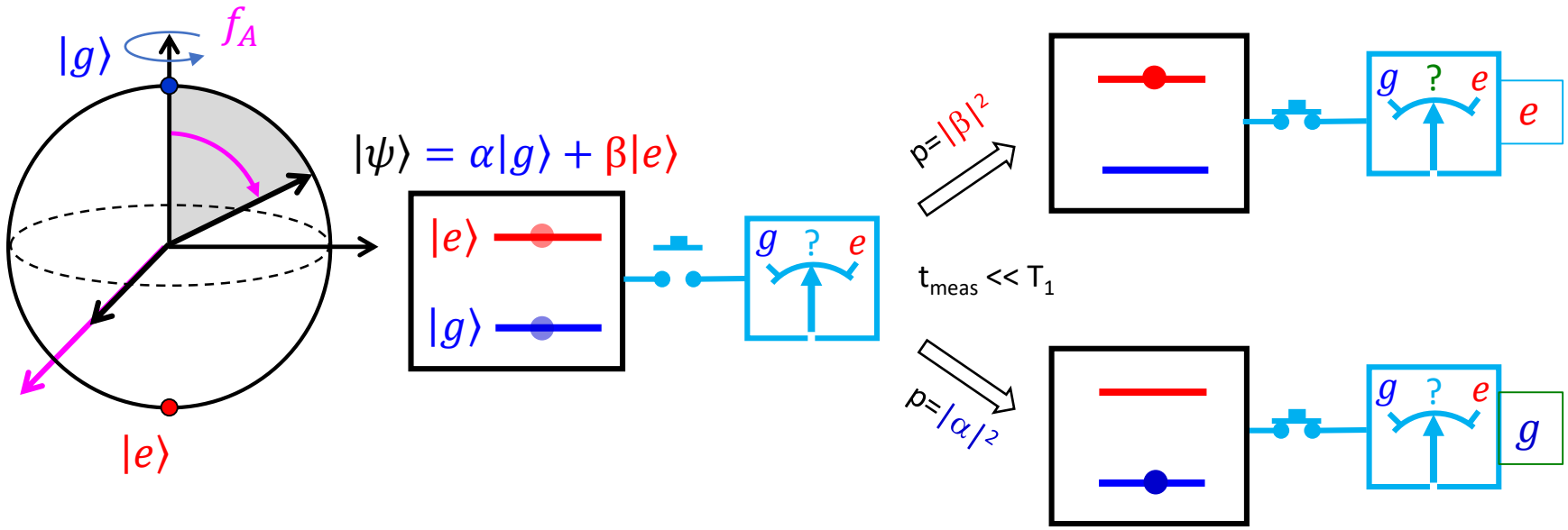
Rotation on the Bloch sphere induced by classical fields



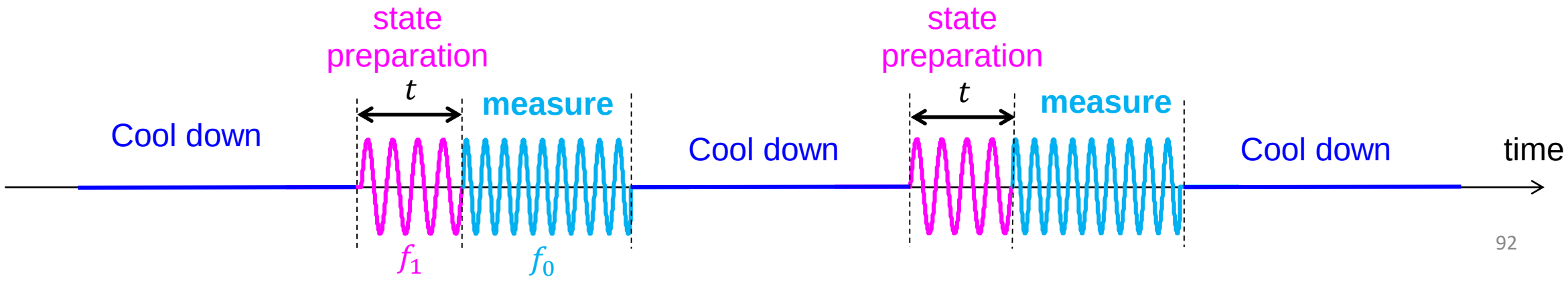
→ state preparation $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$

COHERENT MANIPULATION AT δ=π

Hilbert space of AQD two-level system ($|g\rangle$, $|e\rangle$) → Bloch sphere

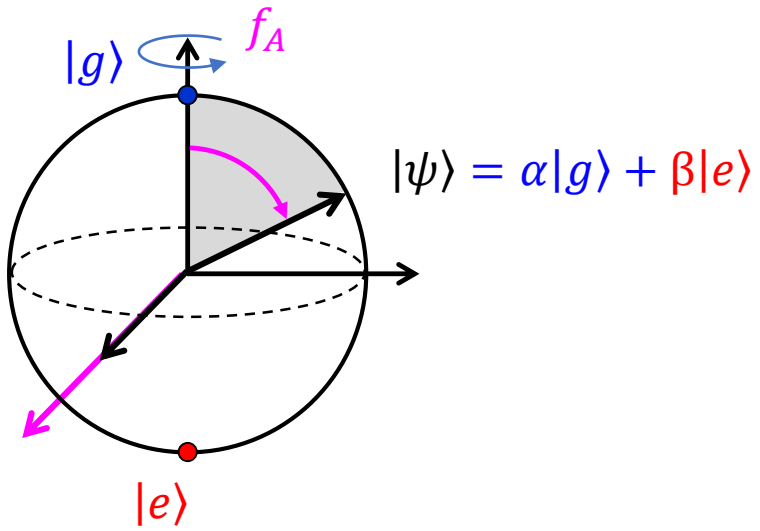


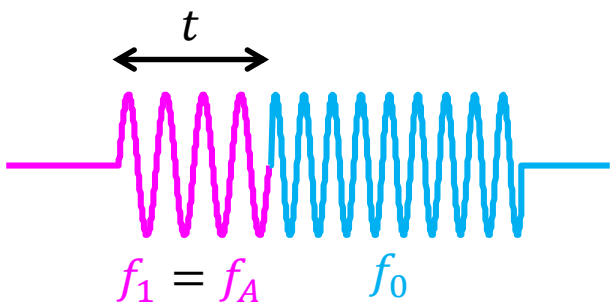
To **measure** $|\alpha|^2$; $|\beta|^2$
repeat
same experimetal sequence

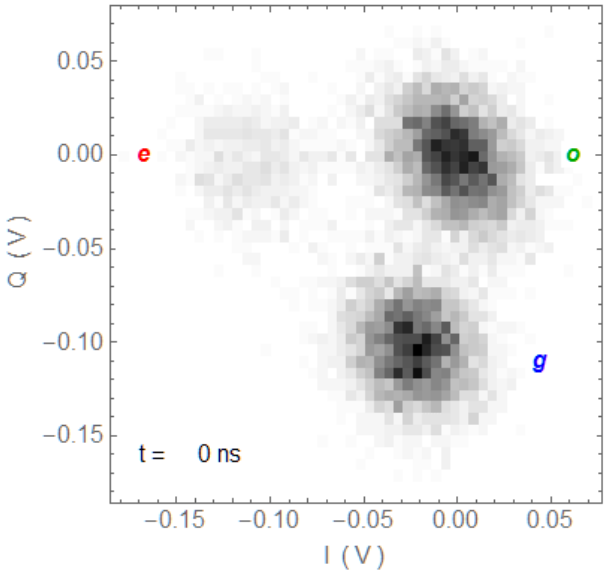


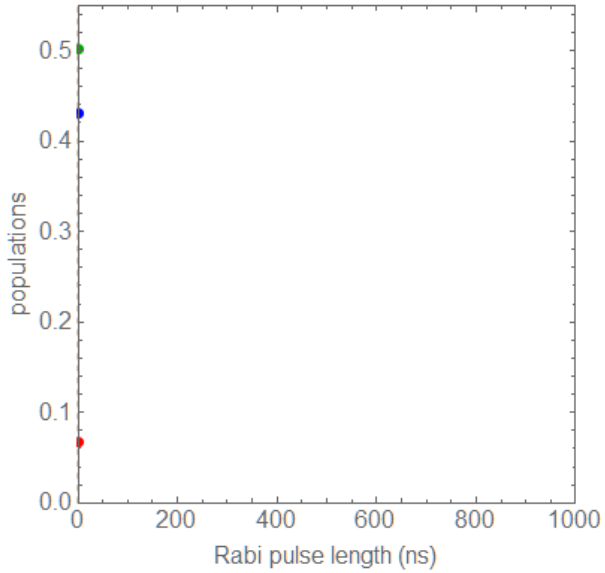
COHERENT MANIPULATION AT δ=π

RABI OSCILLATIONS

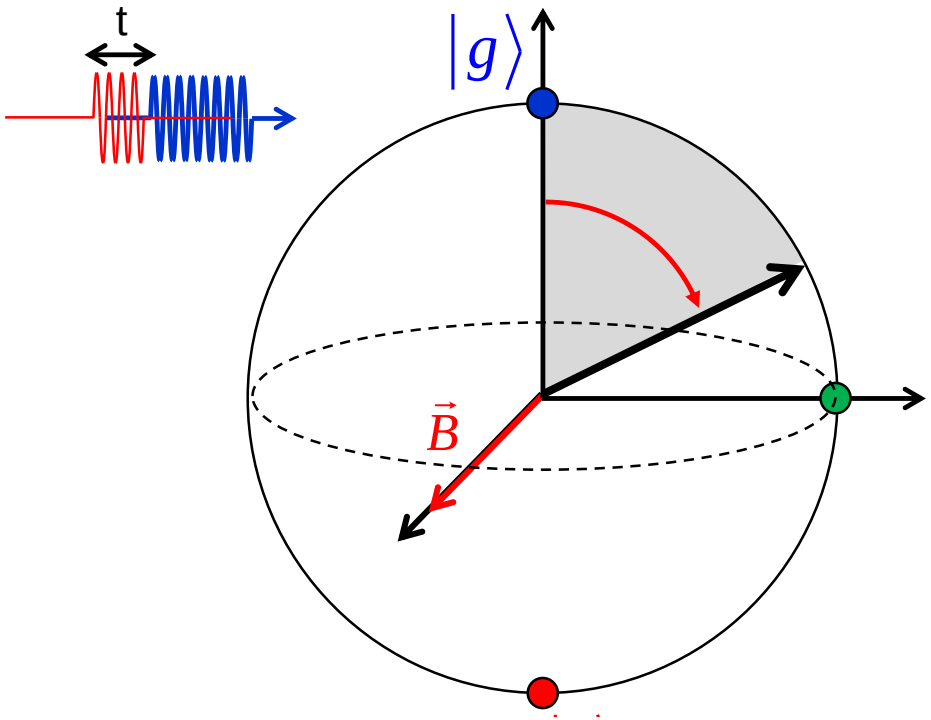


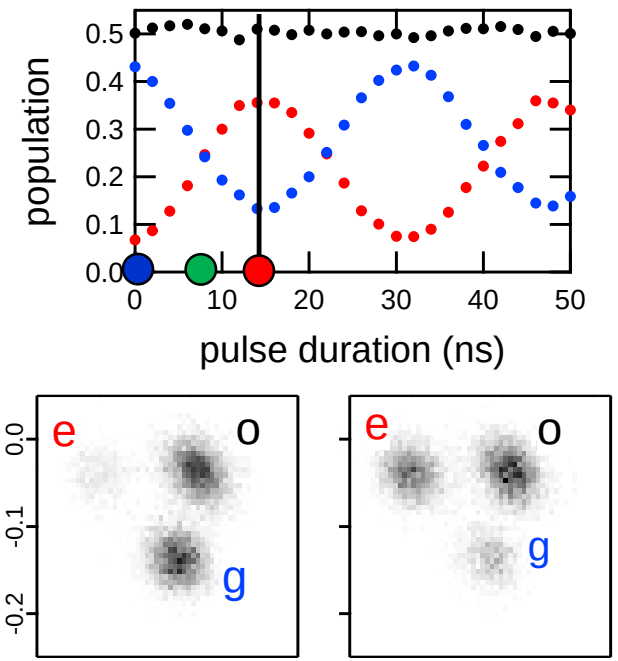


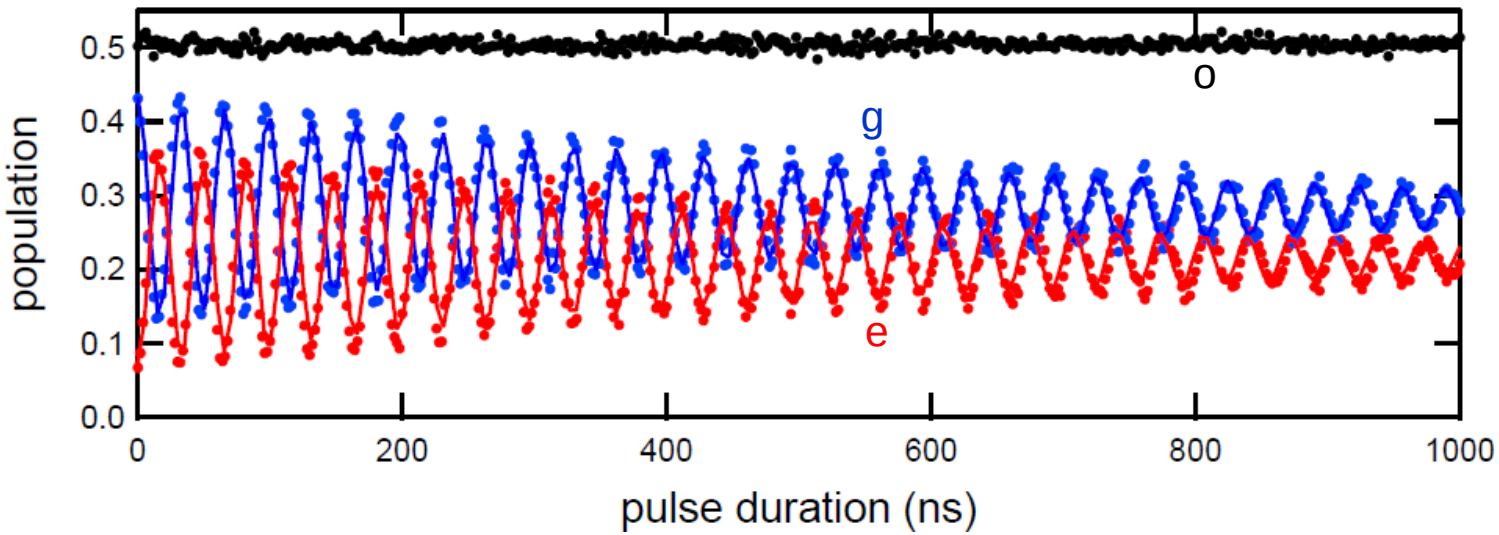




RABI OSCILLATIONS

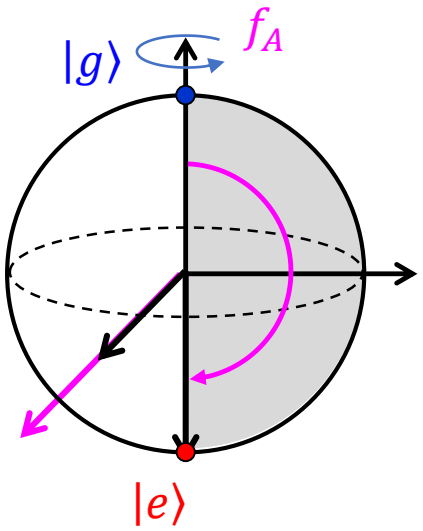


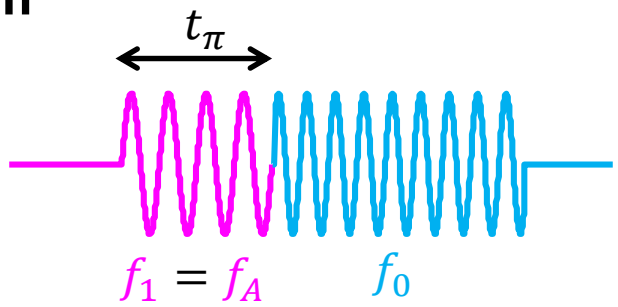


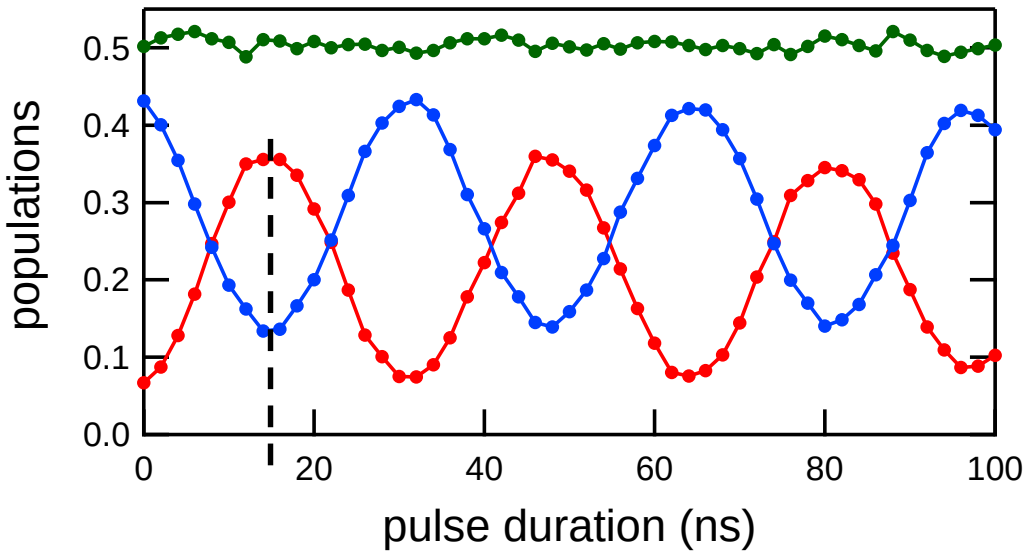


COHERENT MANIPULATION AT δ=π

RABI OSCILLATIONS to initialize the state of the system

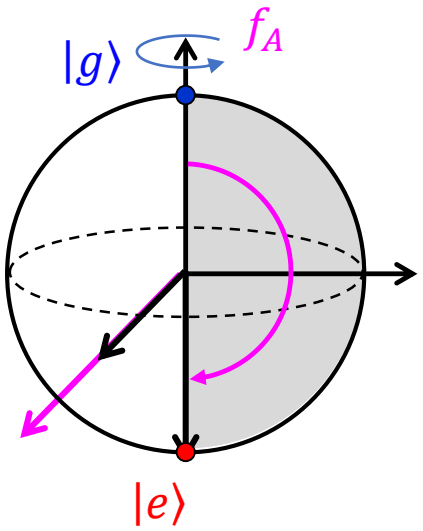




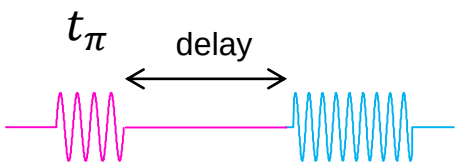


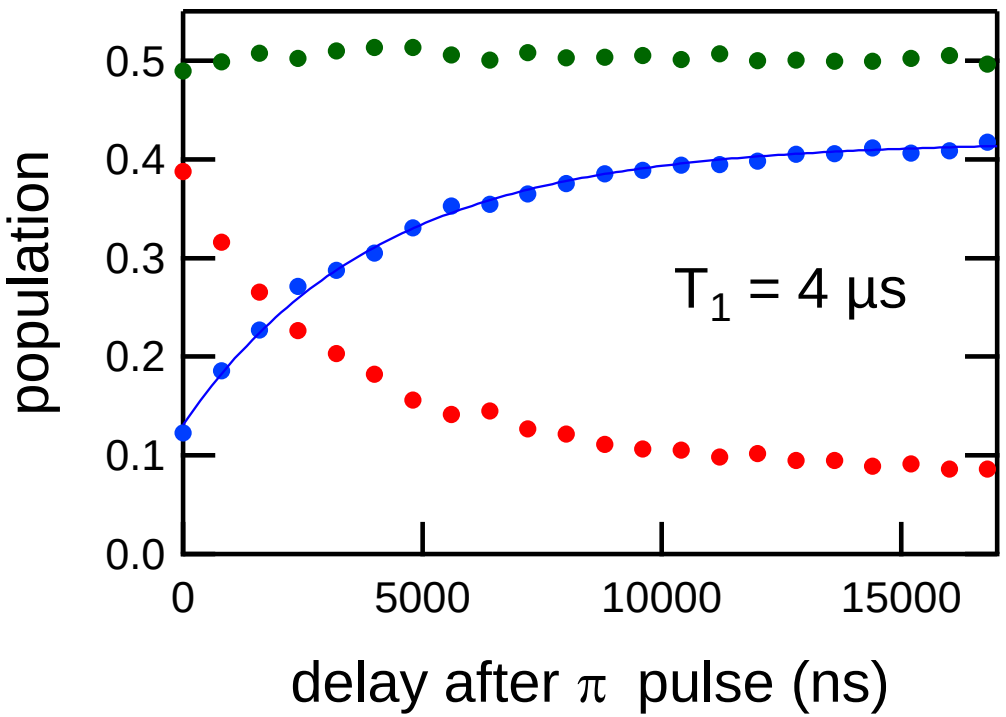
π- Pulse

LIFE TIME (T_1)

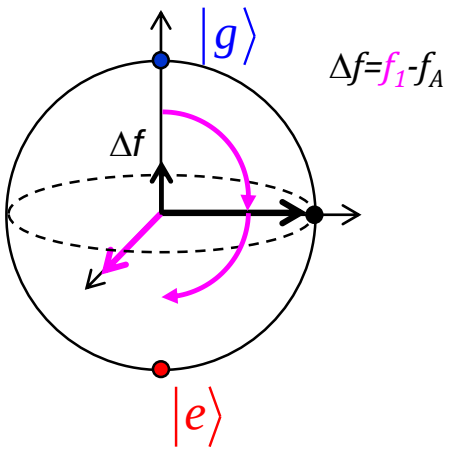


RELAXATION (T1)

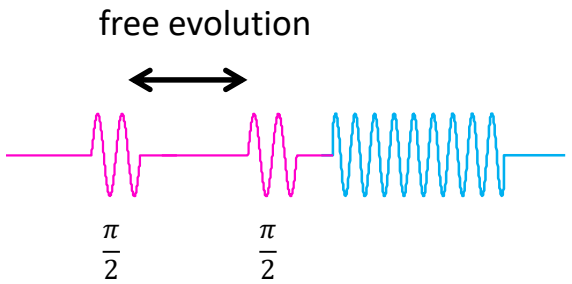


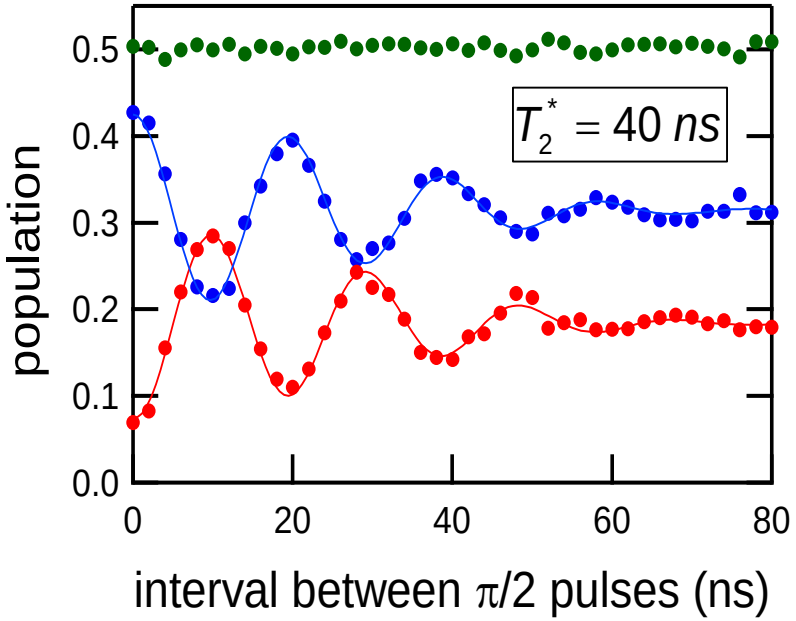


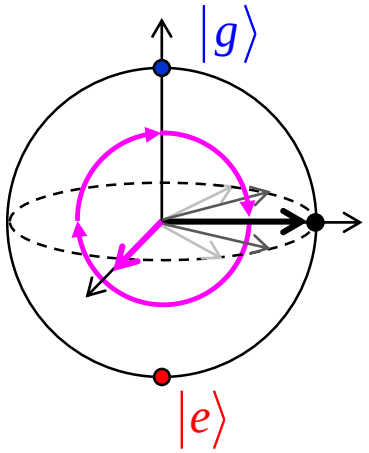
COHERENCE TIME



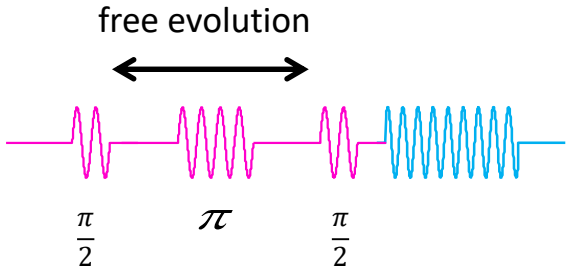
RAMSEY FRINGES

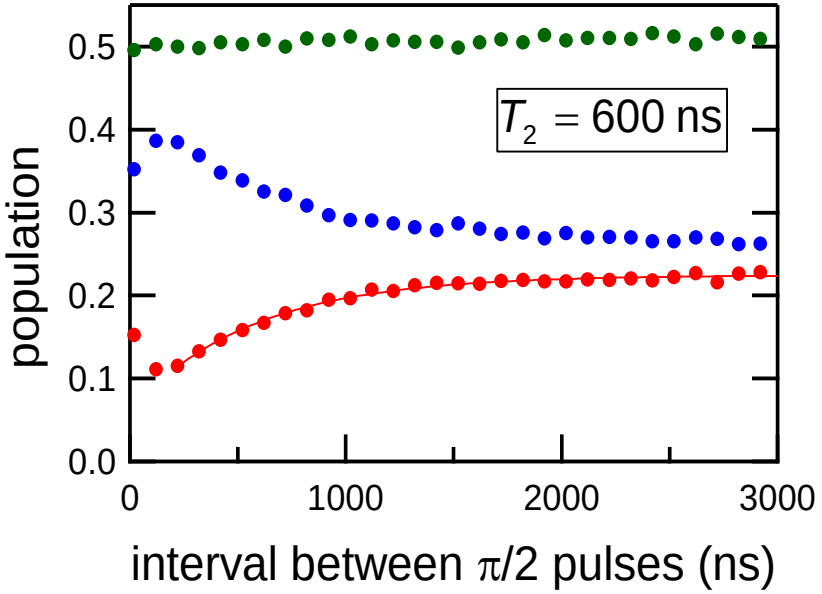






HAHN ECHO



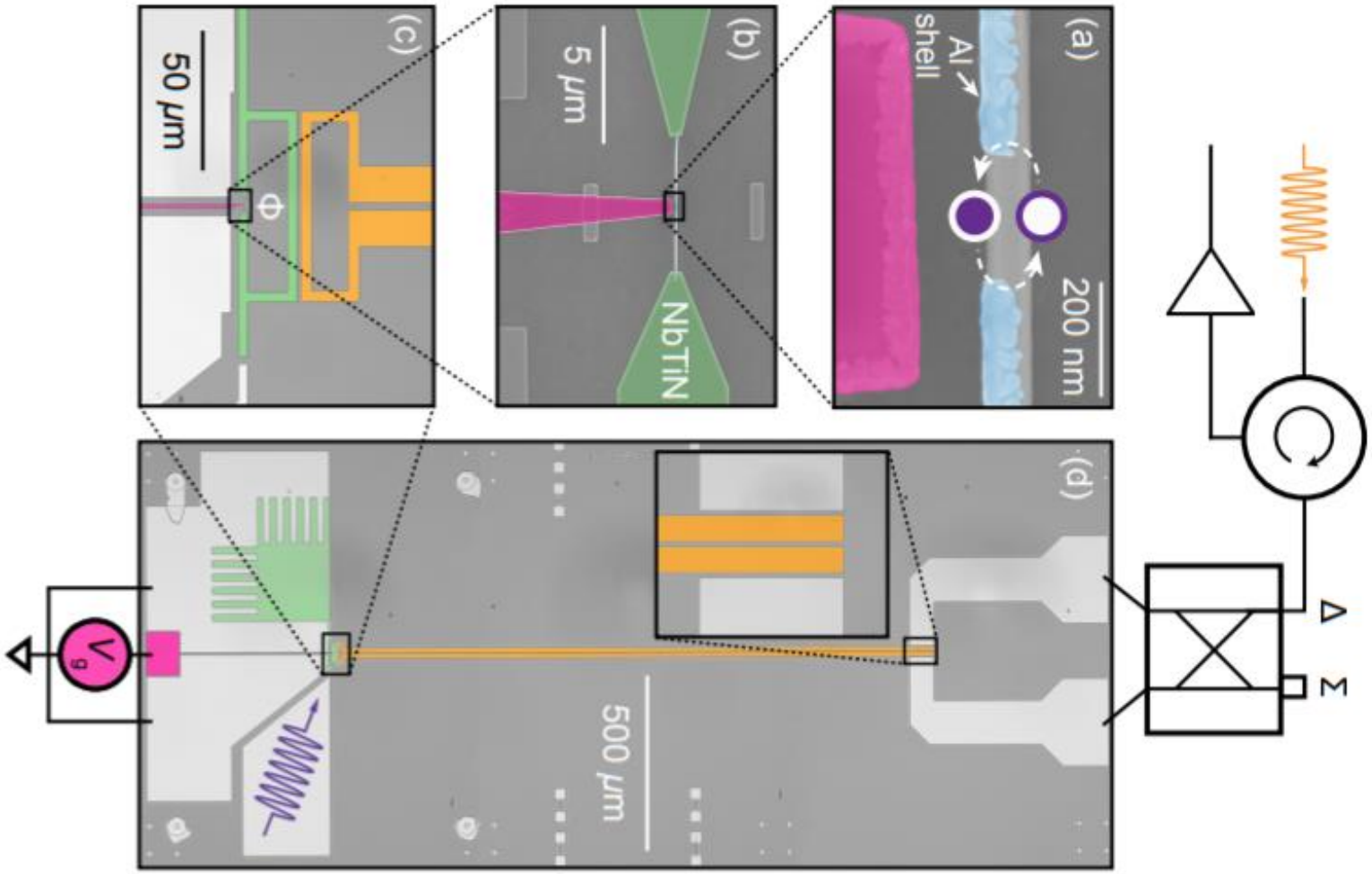


C. Janvier *et al.*, Science **349**, 1199 (2015)

MICROWAVE SPECTROSCOPY OF INAS NANOWIRES

Direct microwave measurement of Andreev-bound-state dynamics in a proximitized semiconducting nanowire

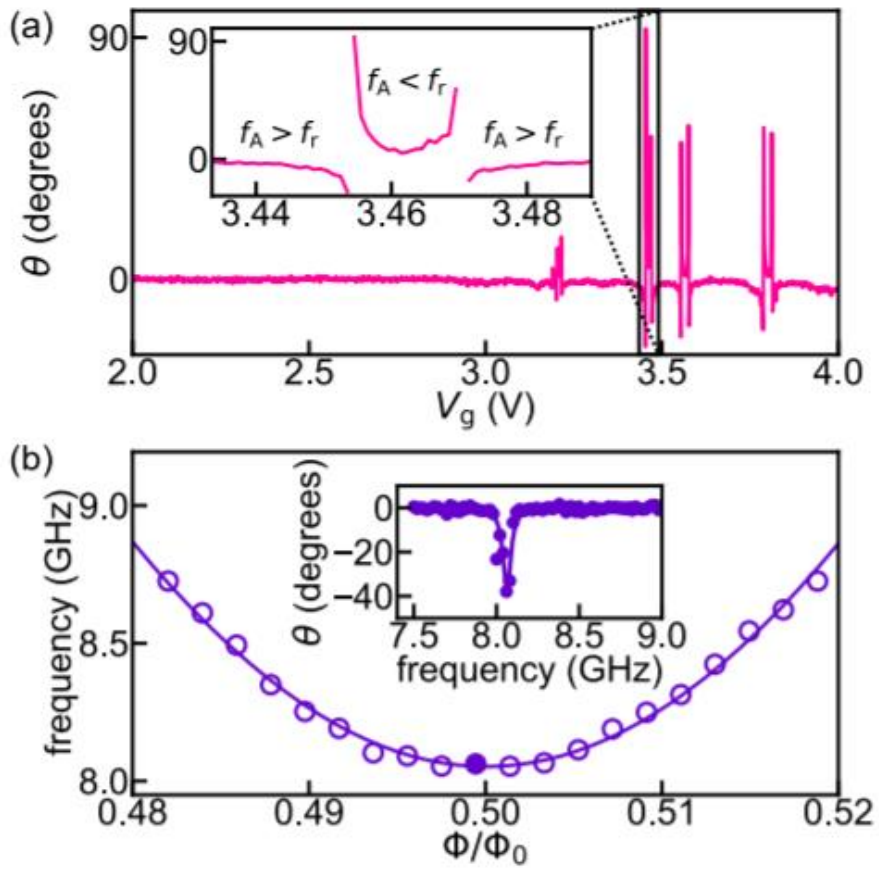
M. Hays,[1,*] G. de Lange,[1,2,3] K. Serniak,[1] D. J. van Woerkom,[2,3]
D. Bouman,[2,3] P. Krogstrup,[4] J. Nygård,[4] A. Geresdi,[2,3] and M. H. Devoret[1,†]

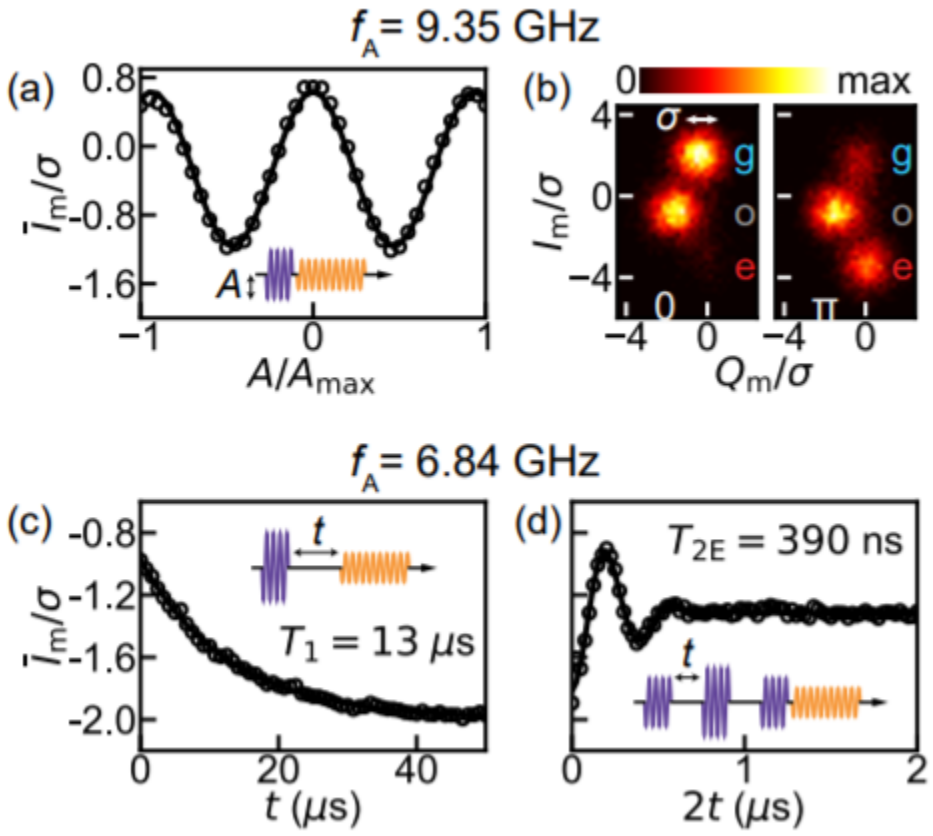


MICROWAVE SPECTROSCOPY OF INAS NANOWIRES

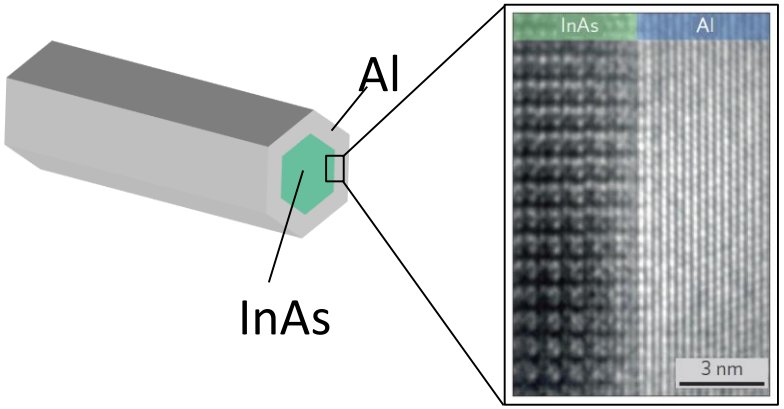
Direct microwave measurement of Andreev-bound-state dynamics in a proximitized semiconducting nanowire

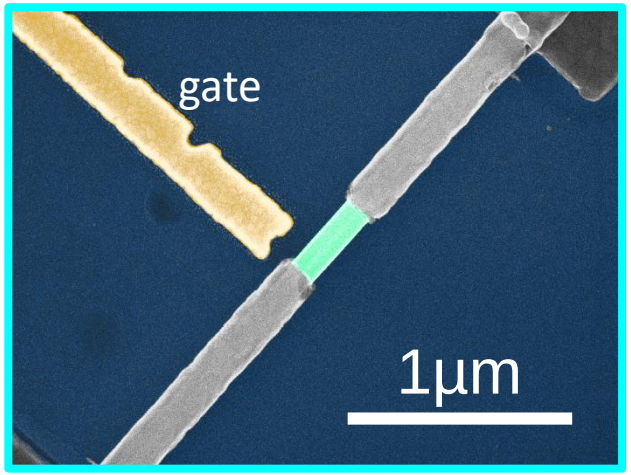
M. Hays,[1,*] G. de Lange,[1,2,3] K. Serniak,[1] D. J. van Woerkom,[2,3]
D. Bouman,[2,3] P. Krogstrup,[4] J. Nygård,[4] A. Geresdi,[2,3] and M. H. Devoret[1,†]





PROBING THE ABSORPTION SPECTRUM OF AN InAs NANOWIRE JUNCTION

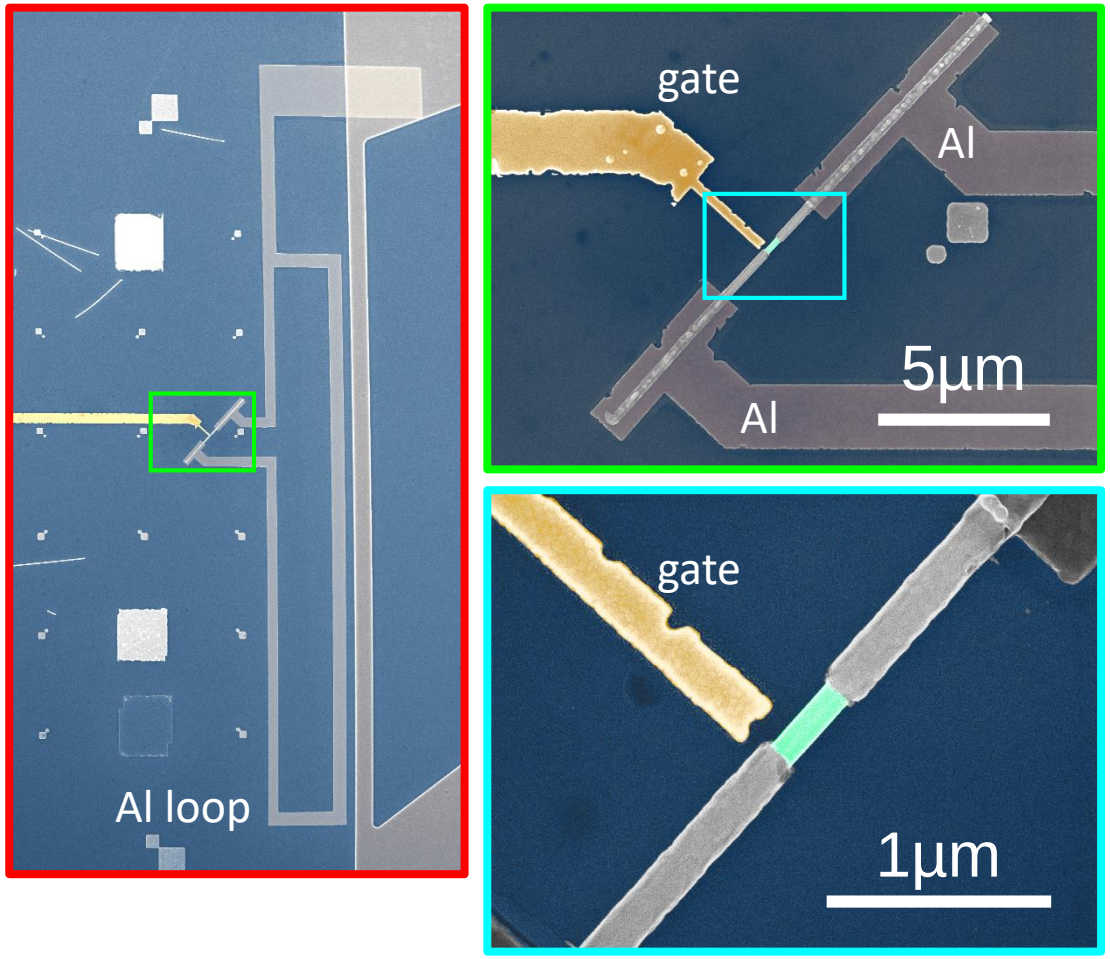




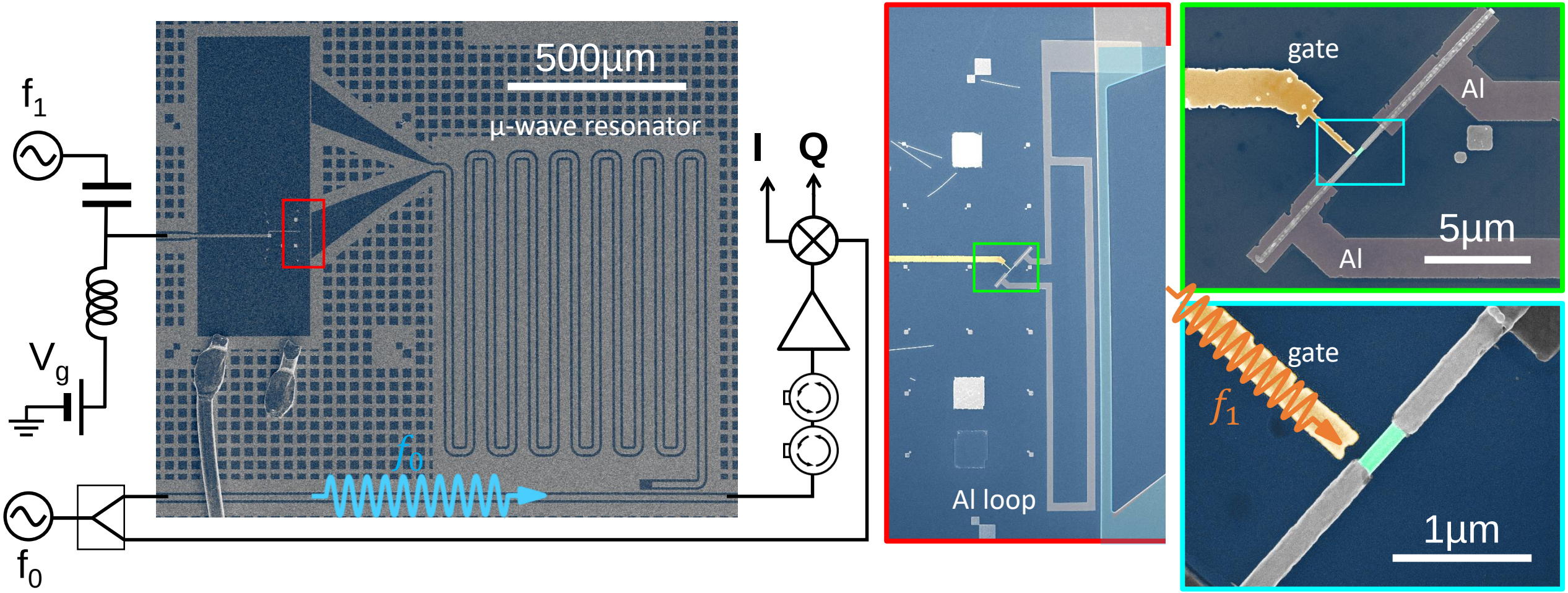
P. Krogstrup

J. Nygård

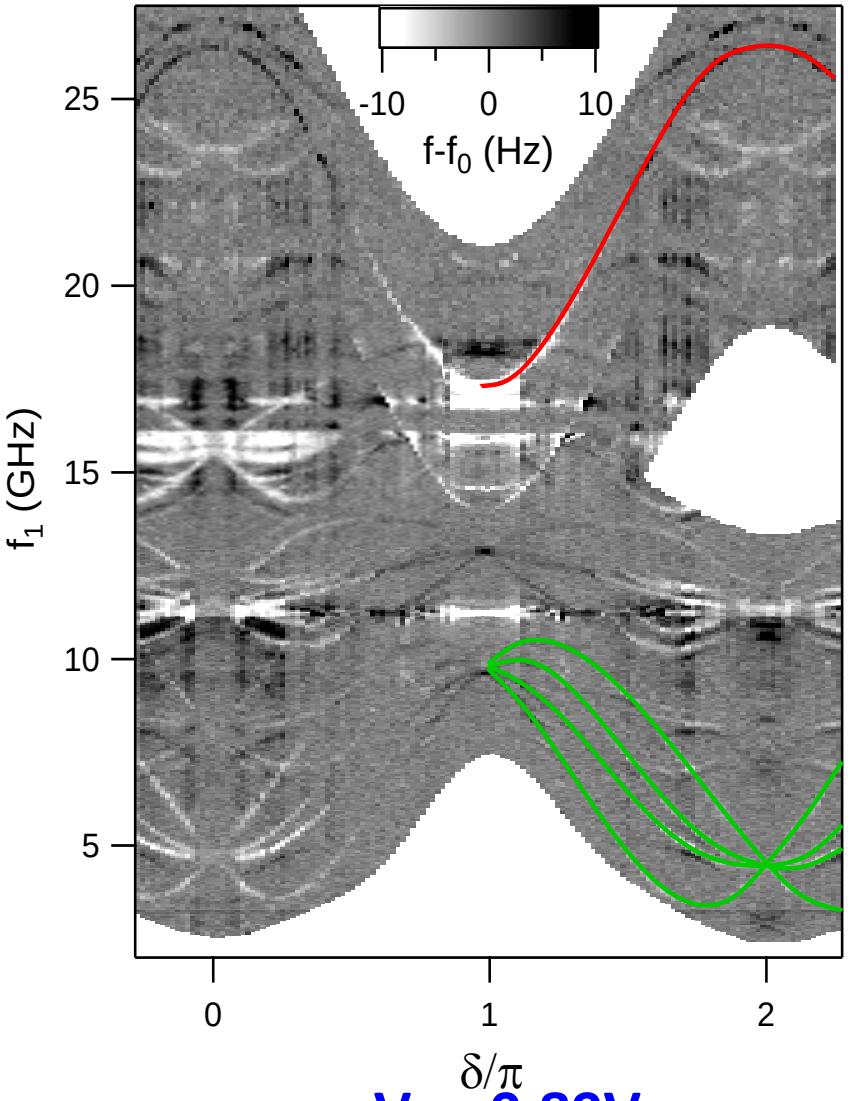
PROBING THE ABSORPTION SPECTRUM OF AN InAs NANOWIRE JUNCTION



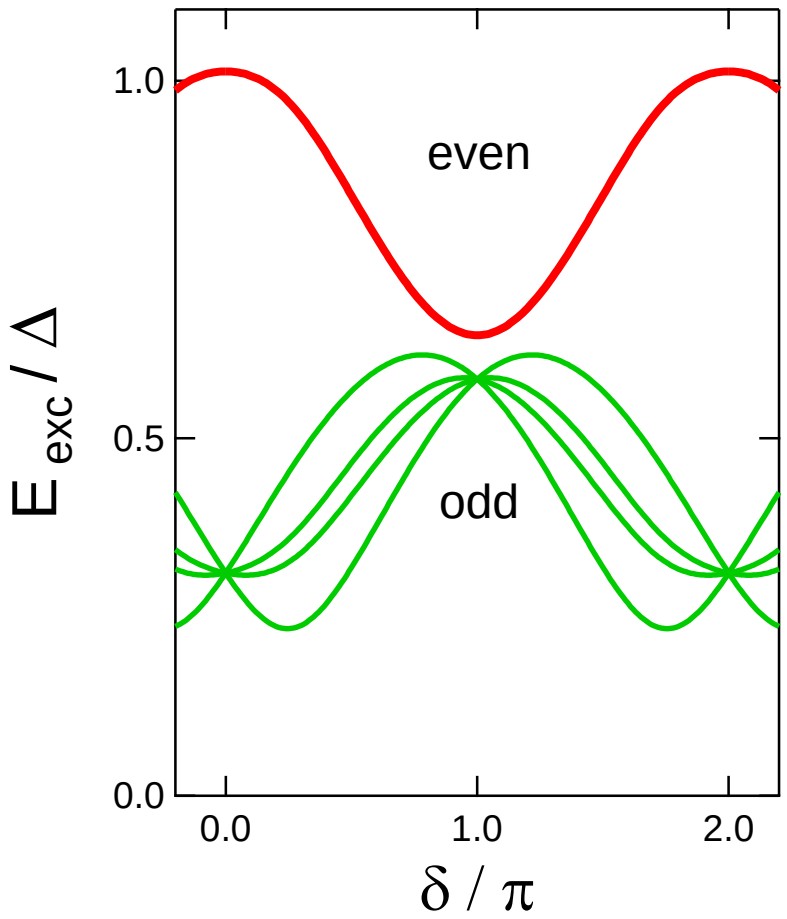
PROBING THE ABSORPTION SPECTRUM OF AN InAs NANOWIRE JUNCTION



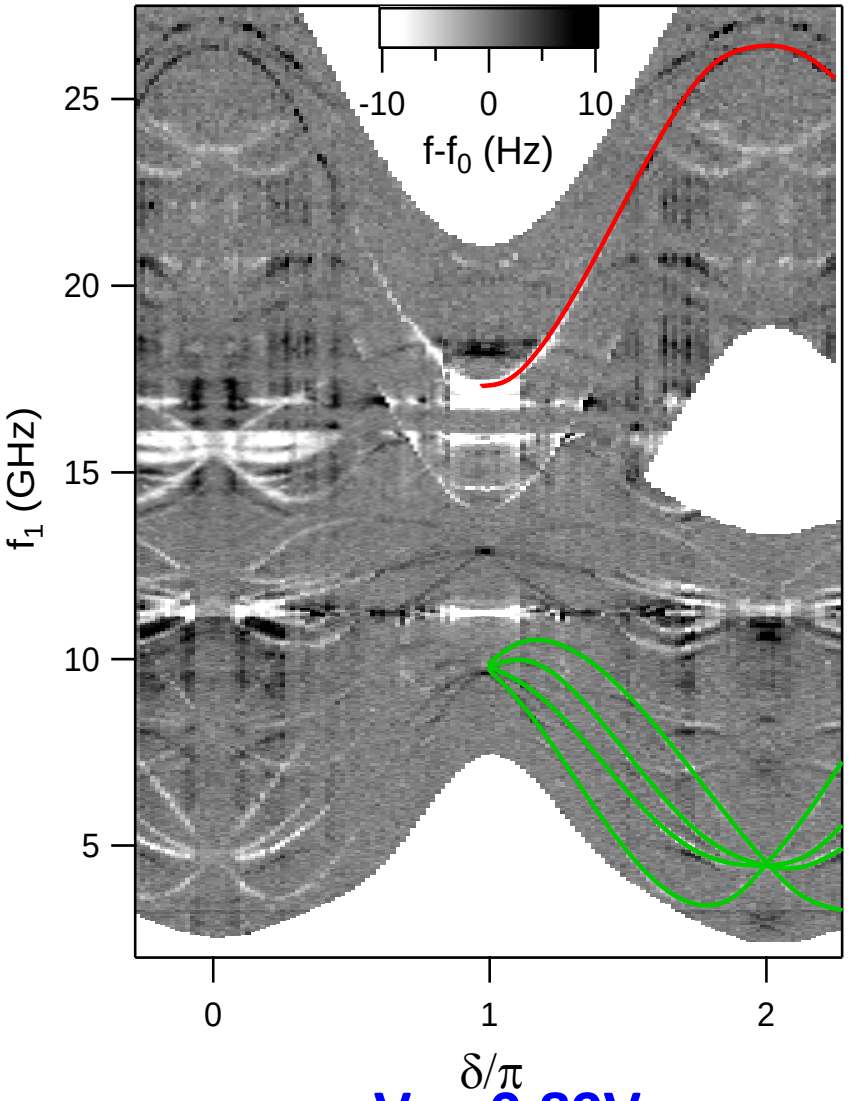
ABSORPTION SPECTRA



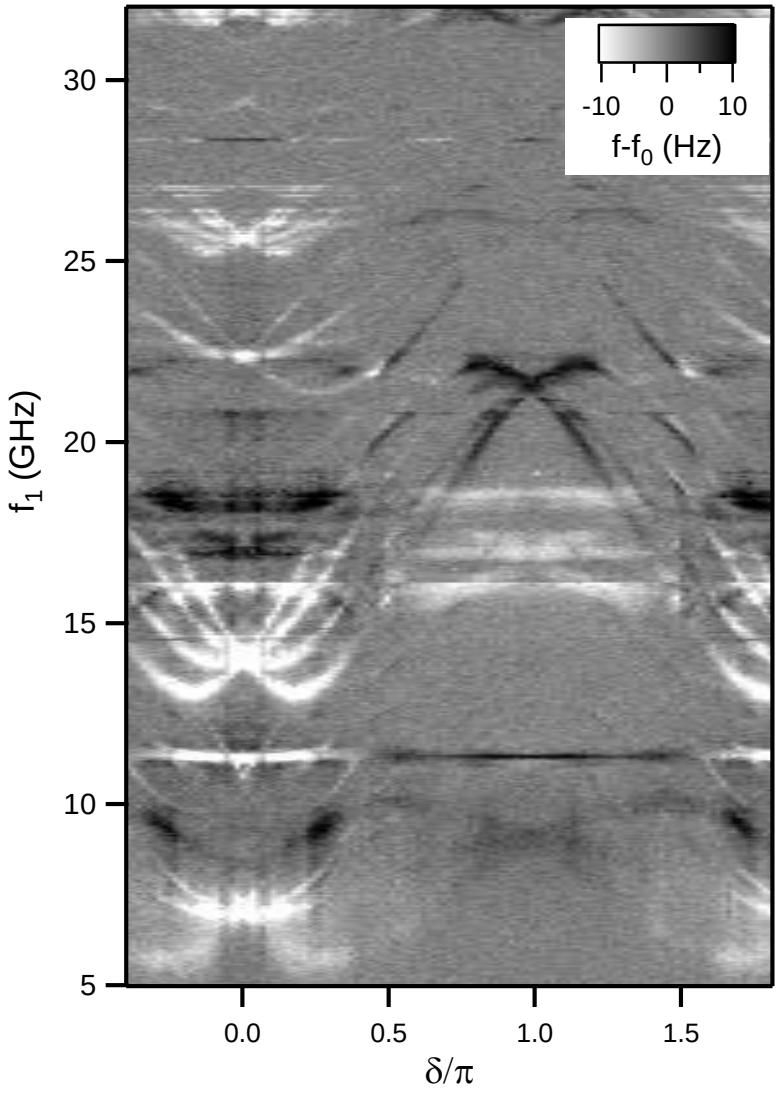
V_G=-0.89V



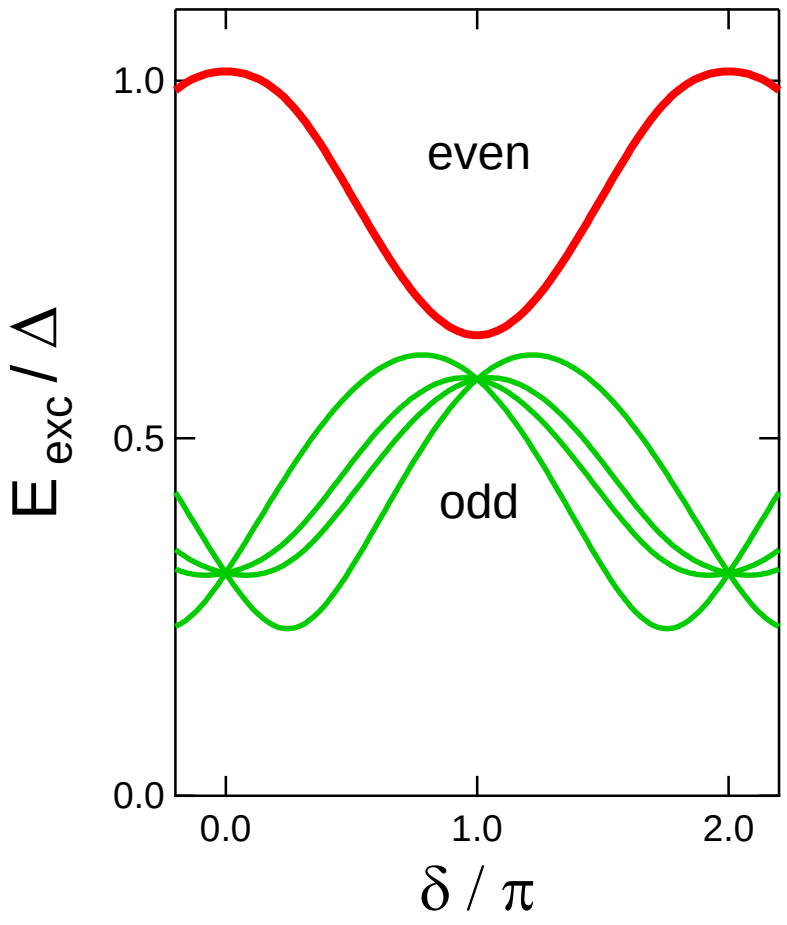
ABSORPTION SPECTRA



V_G=-0.89V



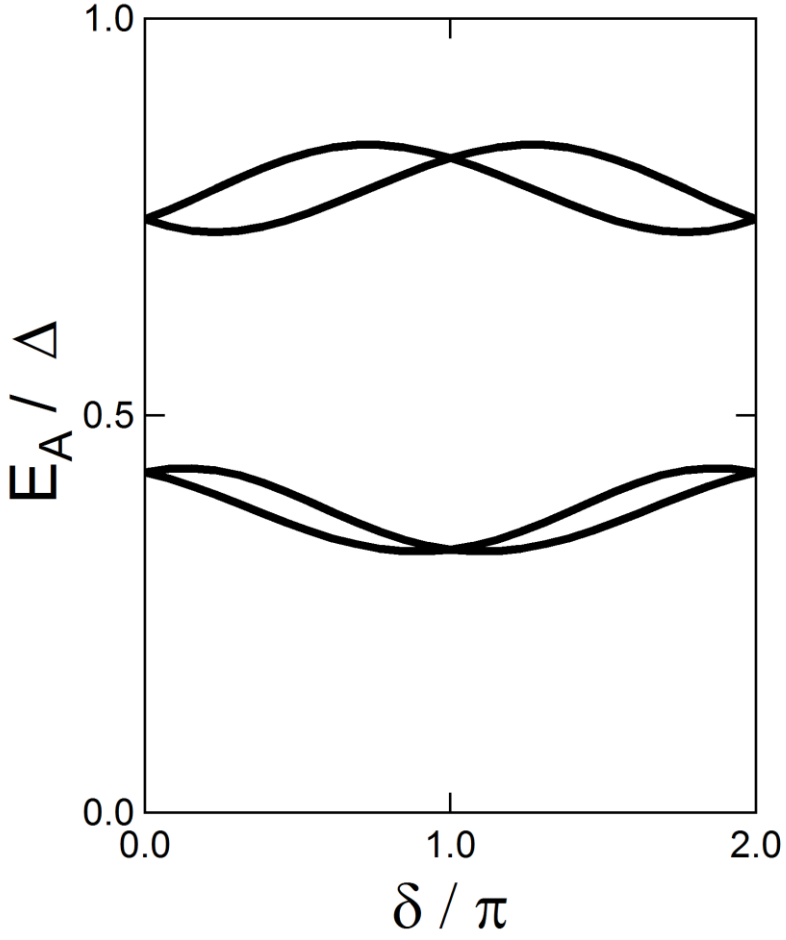
V_G=0.5V



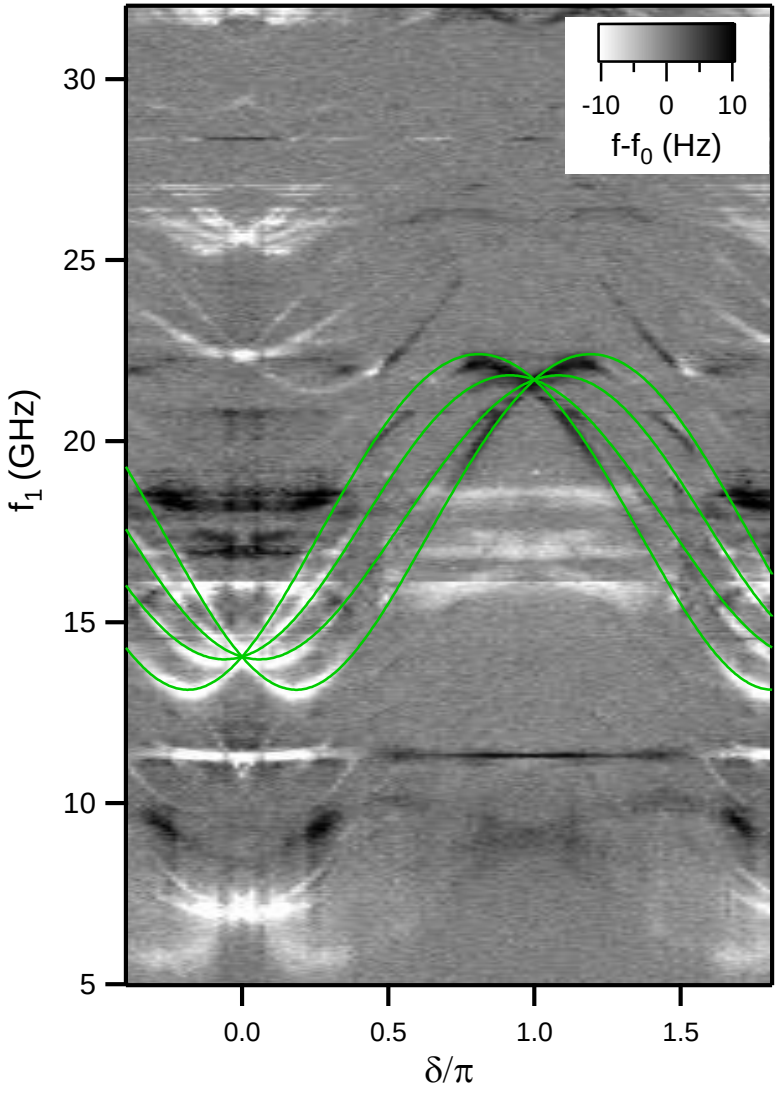
FIT WITH THEORY

$\lambda_1 = 1.3 \quad \tau = 0.295$

$\lambda_2 = 2.3 \quad x_r = 0.525$



Andreev bound states



Absorption spectrum

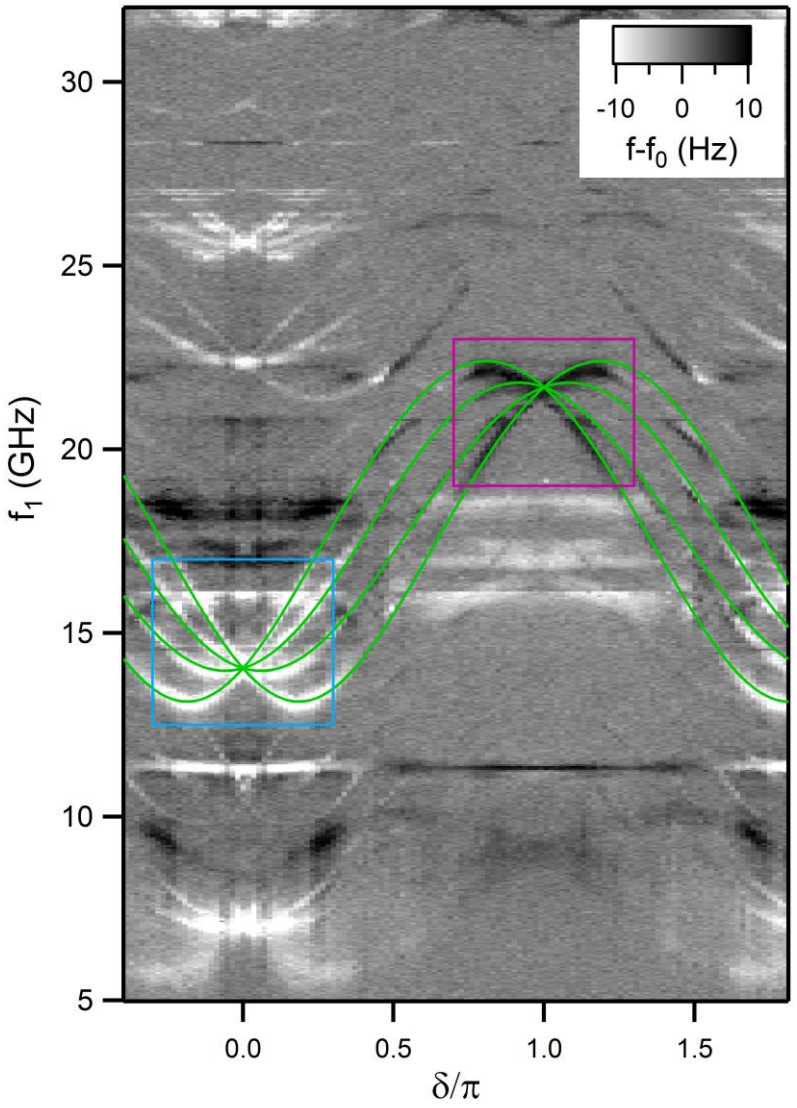
Theory for *odd* transitions

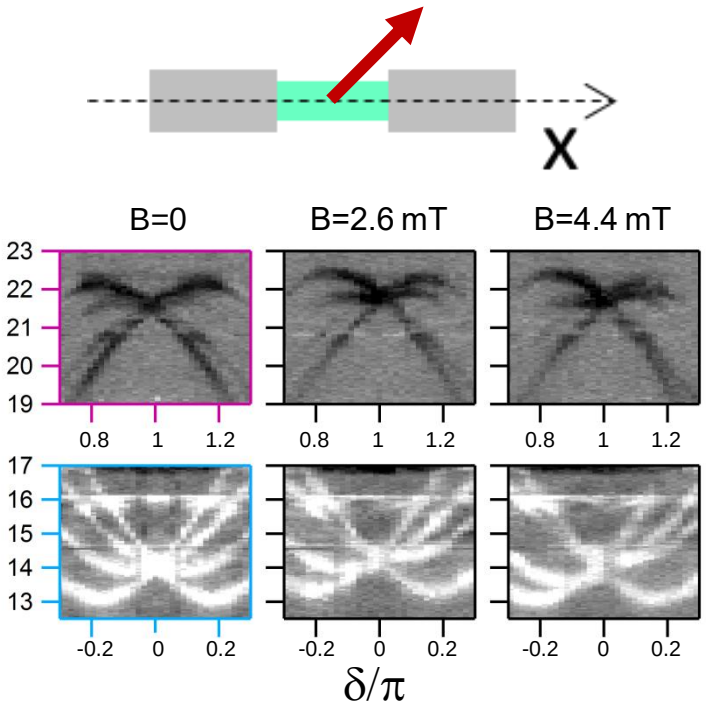
$$\tau\cos((\lambda_1-\lambda_2)\varepsilon \pm \delta) + (1-\tau)\cos((\lambda_1+\lambda_2)\varepsilon x_r) = \cos(2\arccos\varepsilon - (\lambda_1+\lambda_2)\varepsilon)$$

A. Levy Yeyati

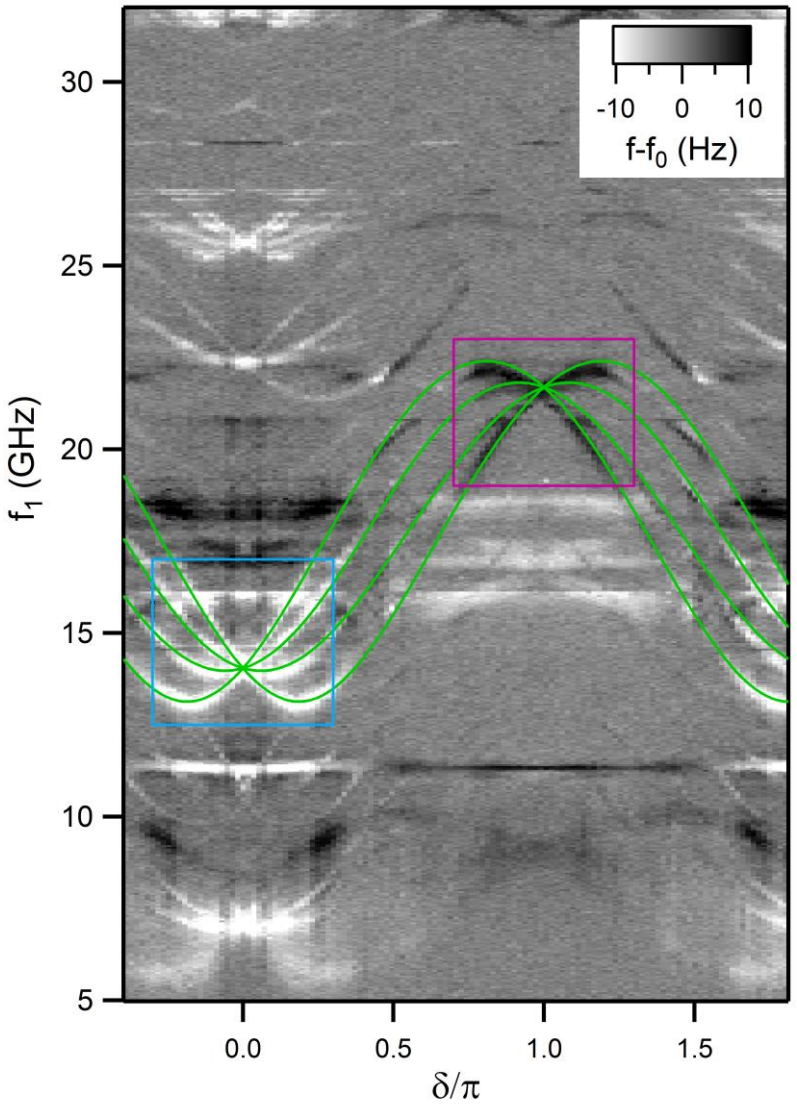
S. Park

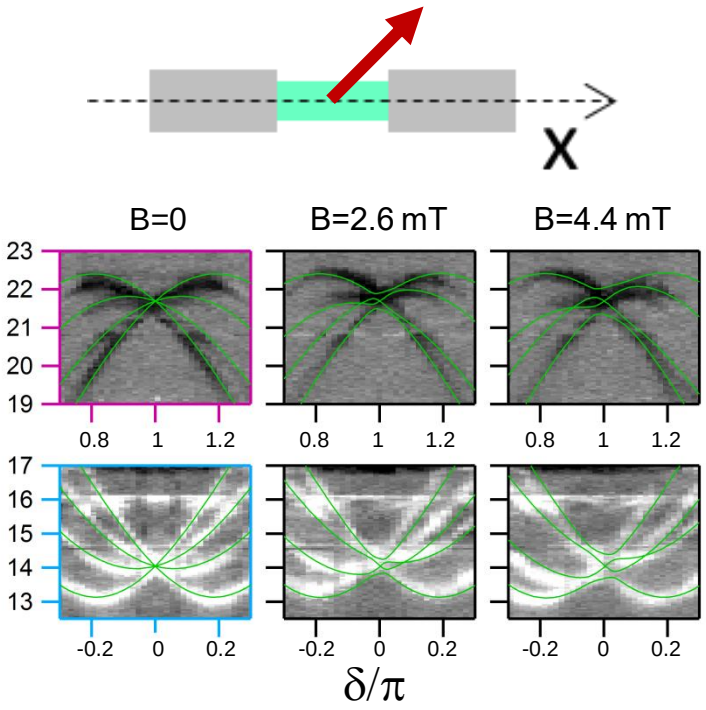
EVOLUTION WITH MAGNETIC FIELD



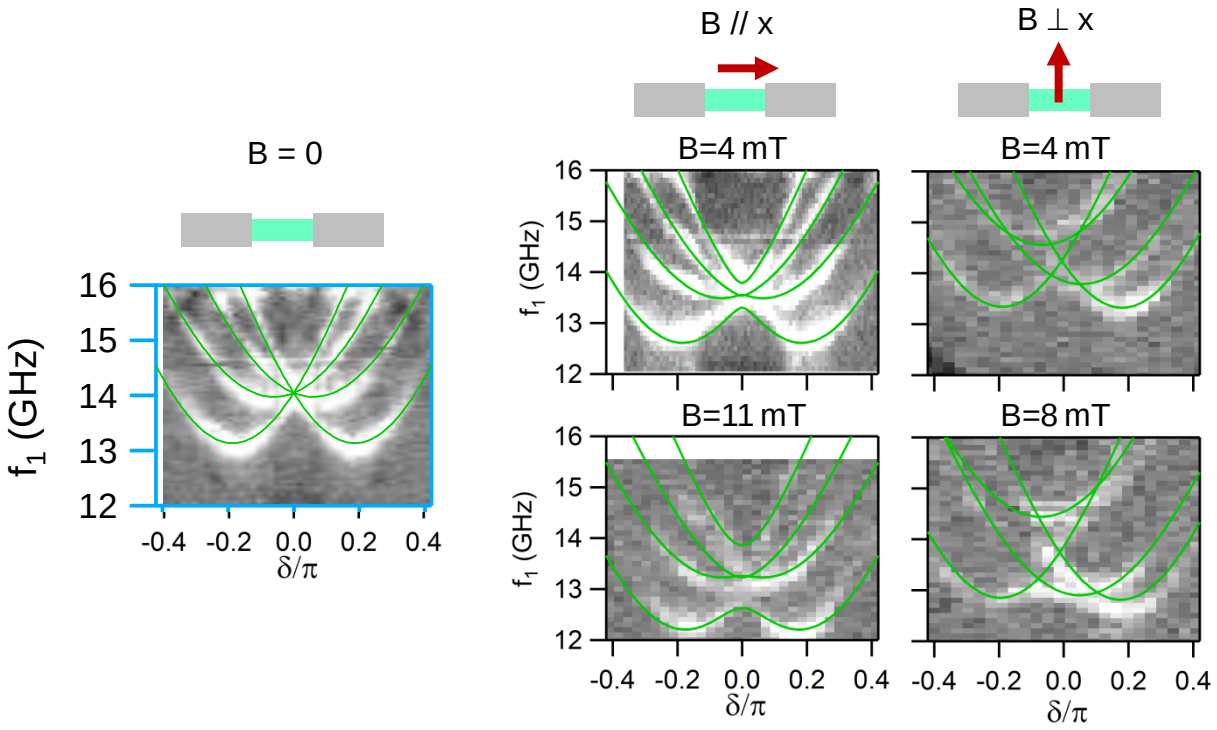


EVOLUTION WITH MAGNETIC FIELD

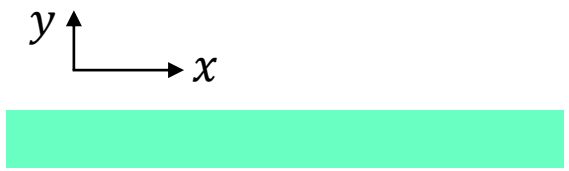




EVOLUTION WITH MAGNETIC FIELD

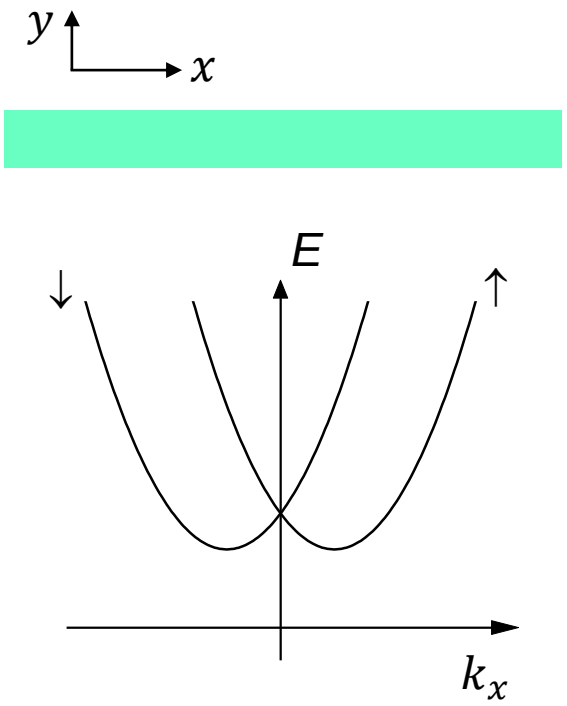


RASHBA SPIN-ORBIT IN A WIRE



$$H = \frac{\vec{p}^2}{2m} + U(y,z) - \alpha(p_x\sigma_y - p_y\sigma_x)$$

RASHBA SPIN-ORBIT IN A 1-D WIRE



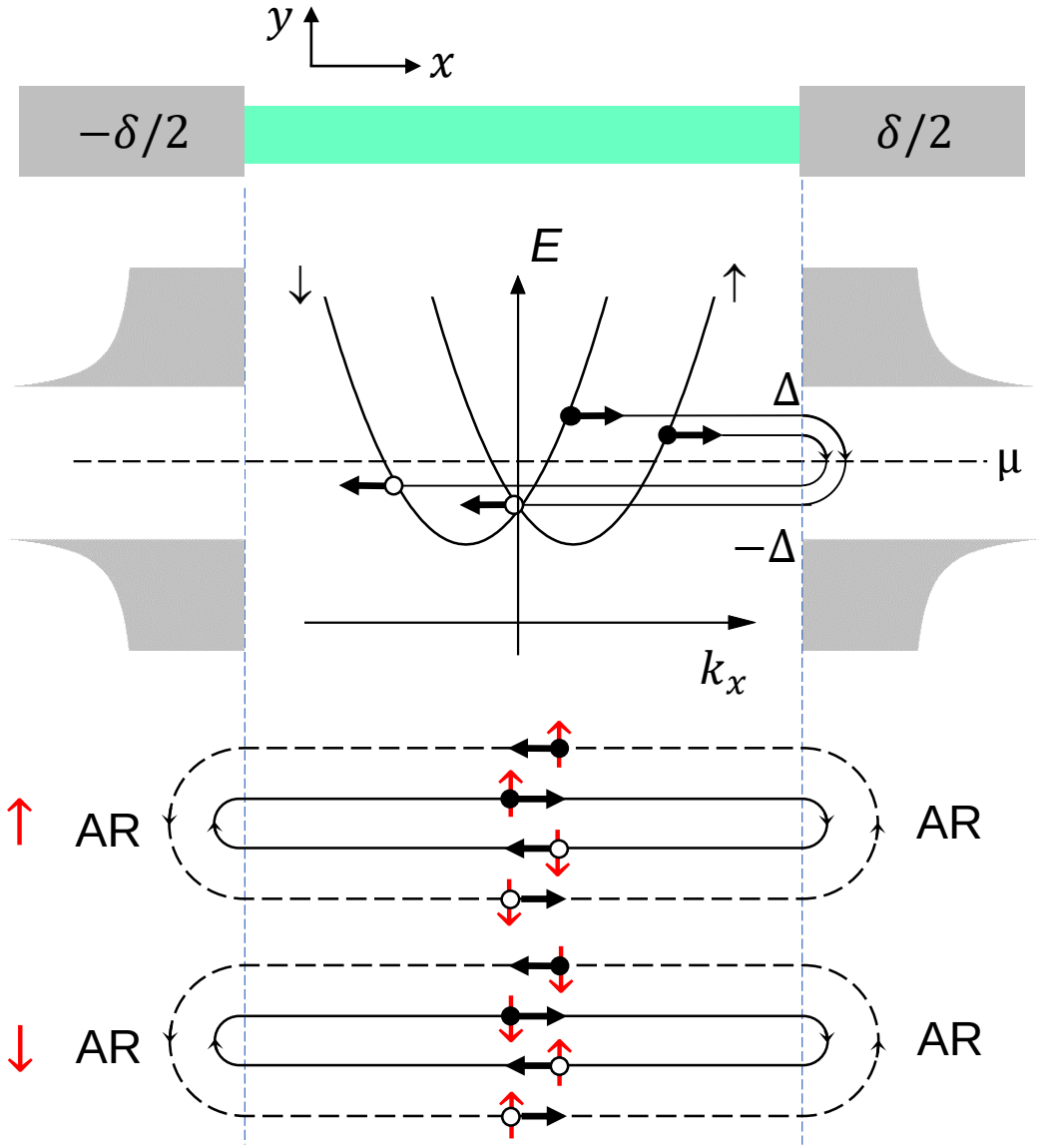
$$H = \frac{\vec{p}^2}{2m} + \cancel{U(y,z)} - \alpha(p_x\sigma_y - \cancel{p_y\sigma_x})$$

$$= \frac{p_x^{\ 2}}{2m} - \alpha p_x \sigma_y$$

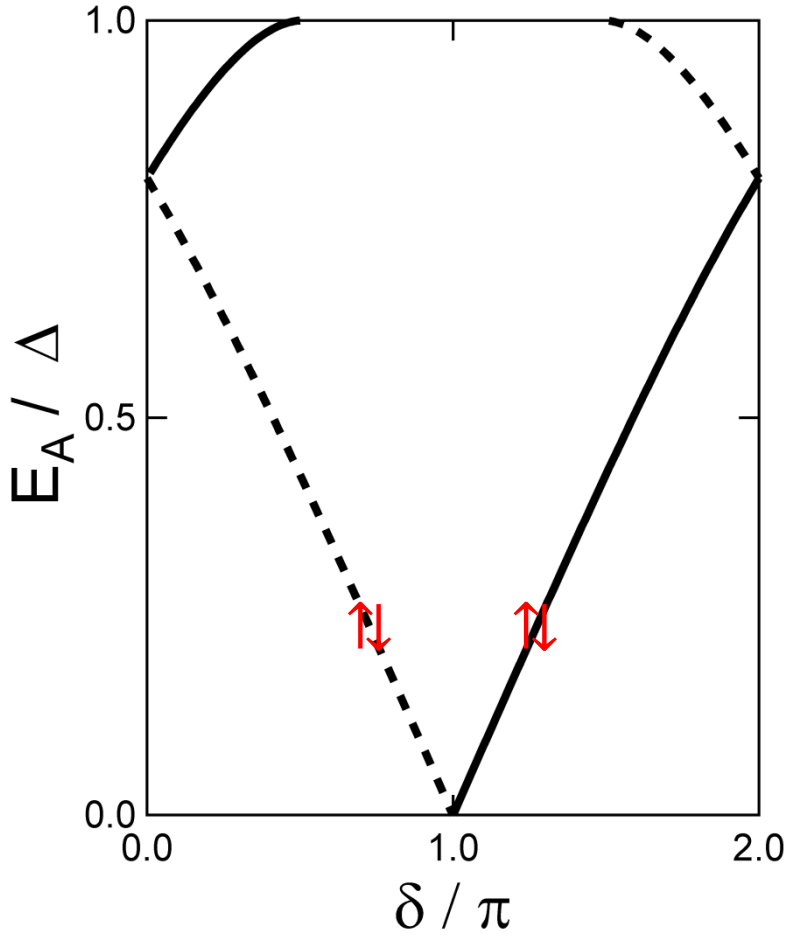
$$= \frac{(p_x - m\alpha\sigma_y)^2}{2m} - \frac{m\alpha^2}{2}$$

➔ spin-split bands

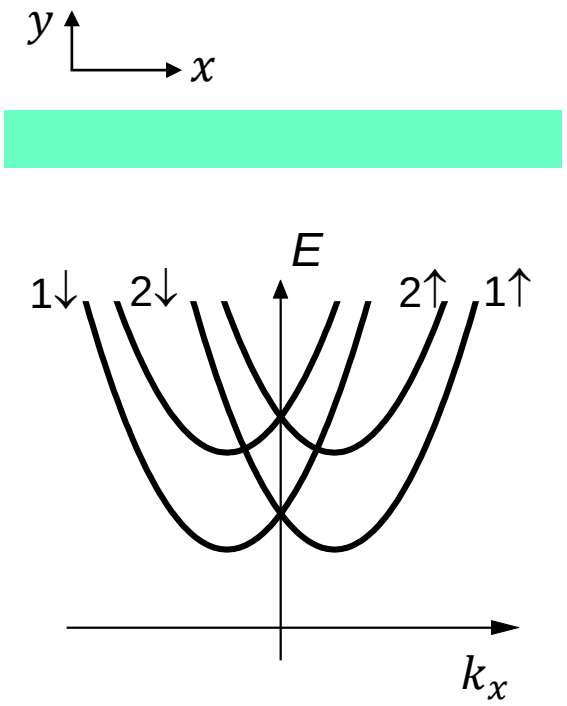
LONG JUNCTION WITH RASHBA SPIN-ORBIT, 1-D



$$\pm\delta - 2\arccos\varepsilon + 2\lambda\varepsilon = 0 \pmod{2\pi}$$



RASHBA SPIN-ORBIT IN A WIRE WITH 2 SUBBANDS

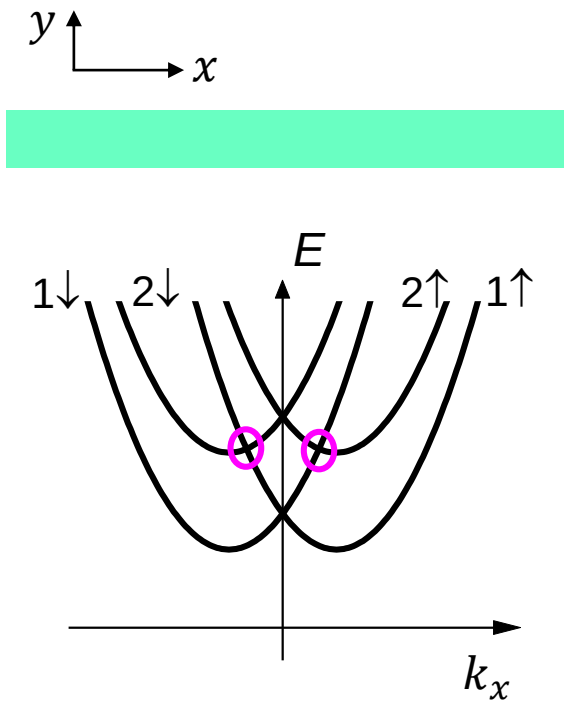


$$H = \frac{\vec{p}^2}{2m} + U(y,z) - \alpha(p_x\sigma_y - p_y\sigma_x)$$

$$= \boxed{\frac{p_x^{\ 2}}{2m} + \frac{p_y^{\ 2}}{2m} - \alpha p_x\sigma_y} + \alpha p_y\sigma_x$$

quantized

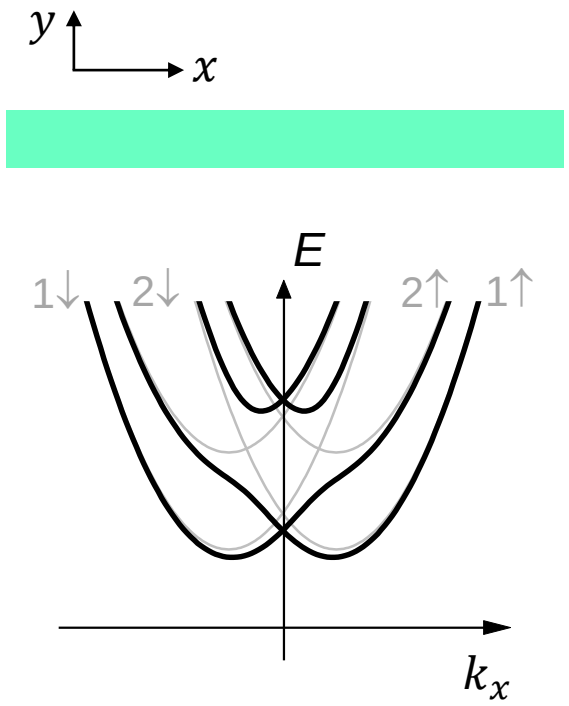
RASHBA SPIN-ORBIT IN A WIRE WITH 2 SUBBANDS



$$H = \frac{\vec{p}^2}{2m} + U(y,z) - \alpha(p_x\sigma_y - p_y\sigma_x)$$

$$= \frac{p_x^{\ 2}}{2m} + \frac{p_y^{\ 2}}{2m} - \alpha p_x\sigma_y + \boxed{\alpha p_y\sigma_x}$$

RASHBA SPIN-ORBIT IN A WIRE WITH 2 SUBBANDS

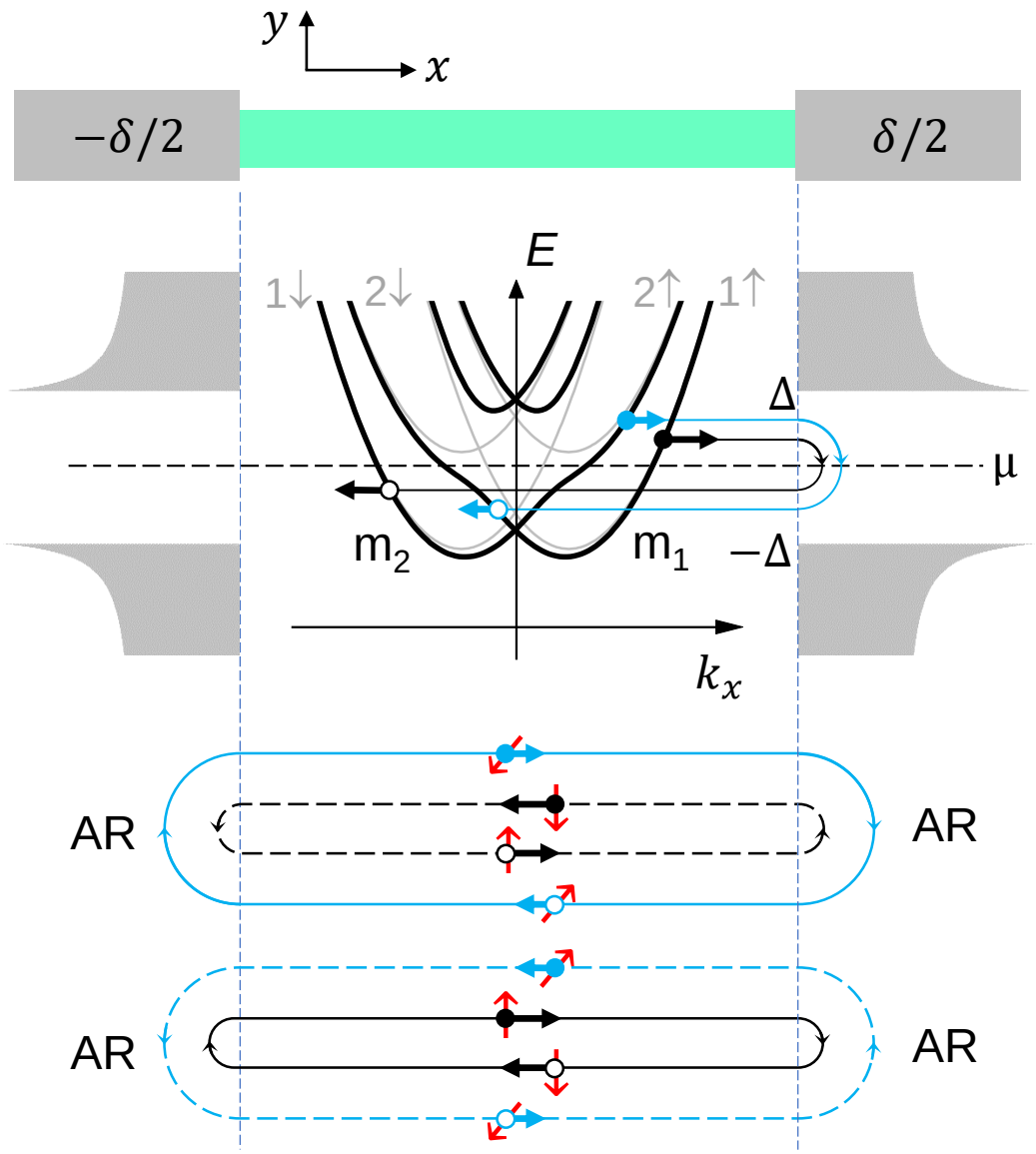


➔ Mixed states, different velocities for each spin

A. Reynoso et al., PRB 86, 214519 (2012)

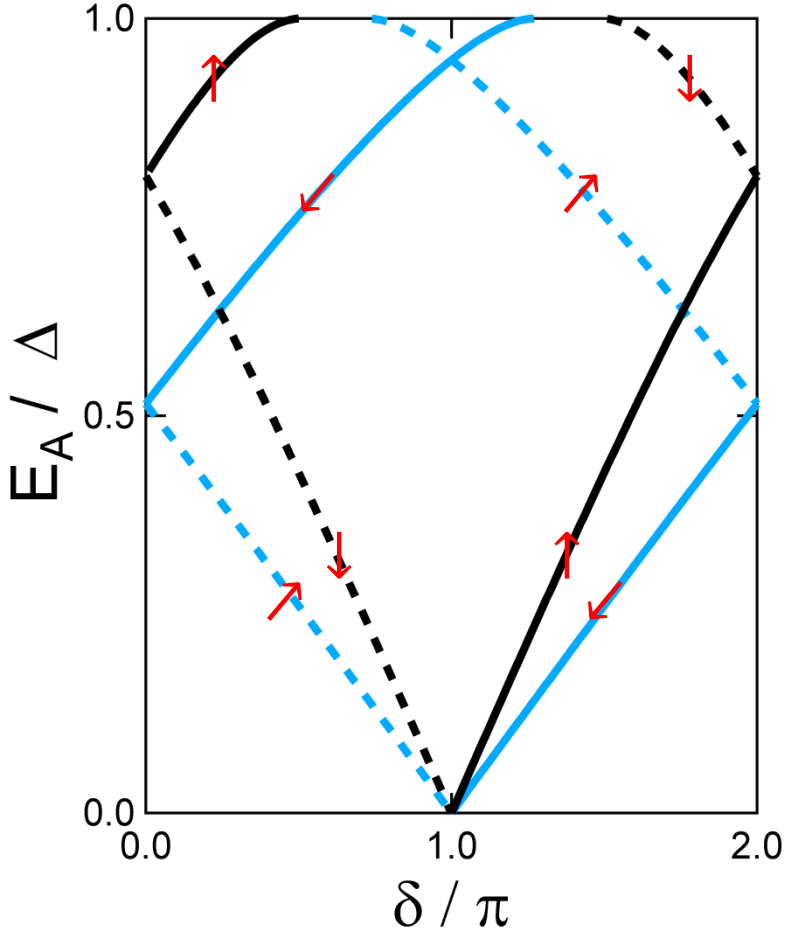
Sunghun Park and A. Levy Yeyati, PRB 96, 125416 (2017)

LONG JUNCTION + 2 SUBBANDS WITH RASHBA SO

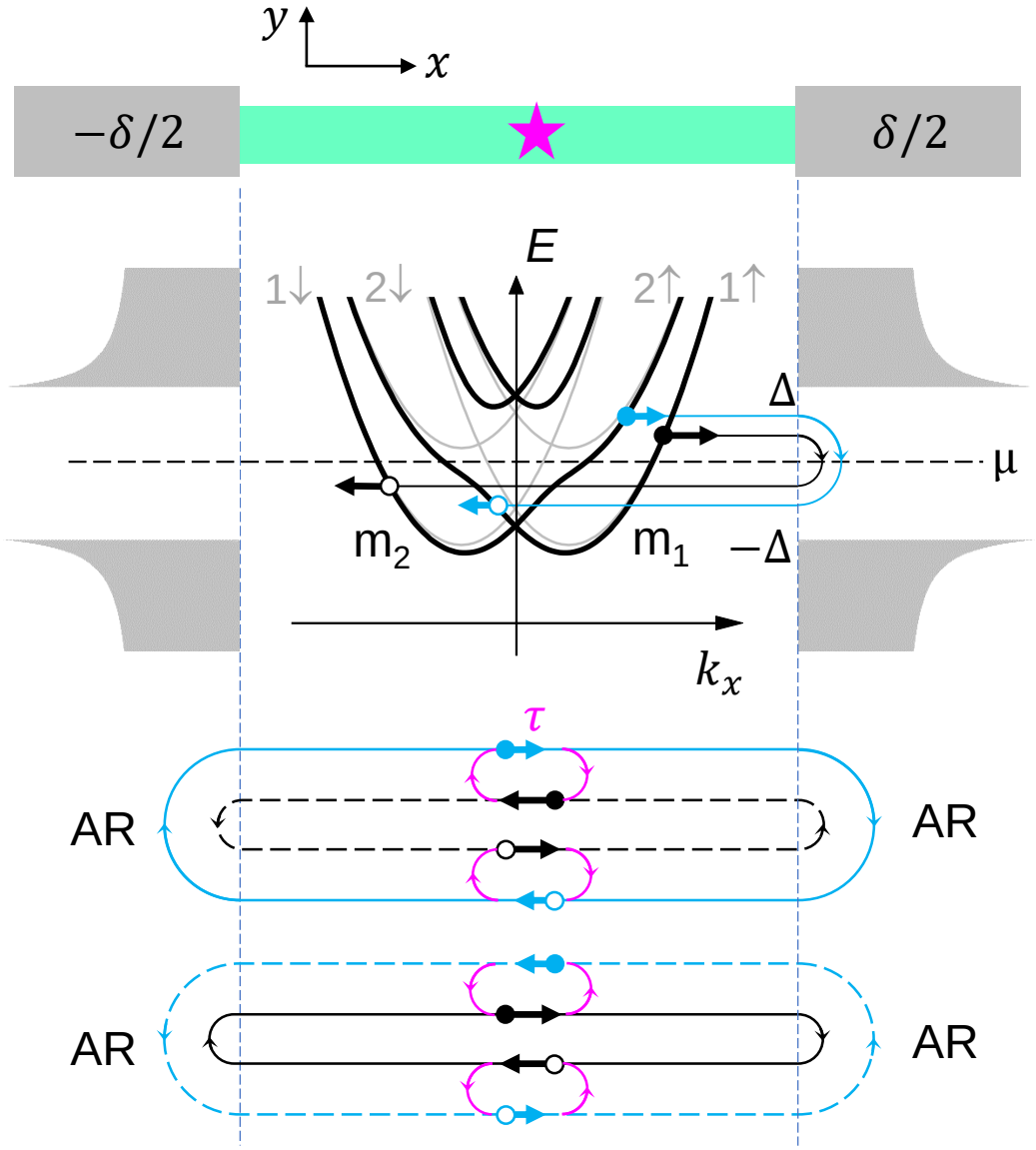


$$\pm\delta - 2\arccos\varepsilon + 2\lambda_i\varepsilon = 0 \pmod{2\pi}$$

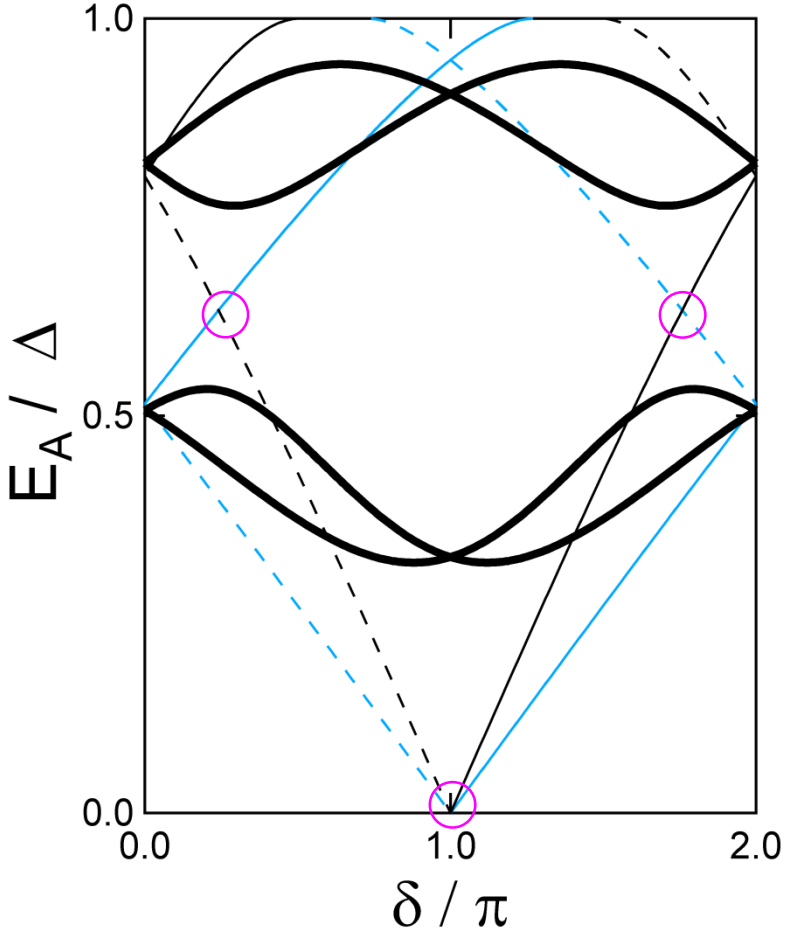
$$\lambda_i = \frac{L}{\hbar V_{Fi}/\Delta}$$



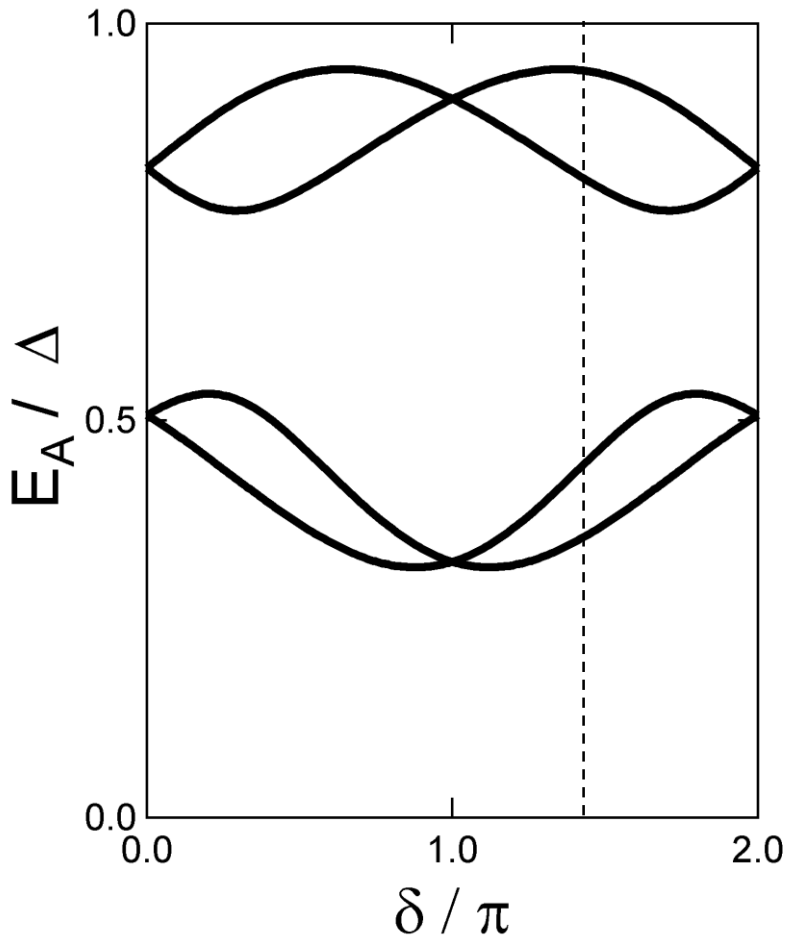
LONG JUNCTION + 2 SUBBANDS WITH SO + SCATTERER



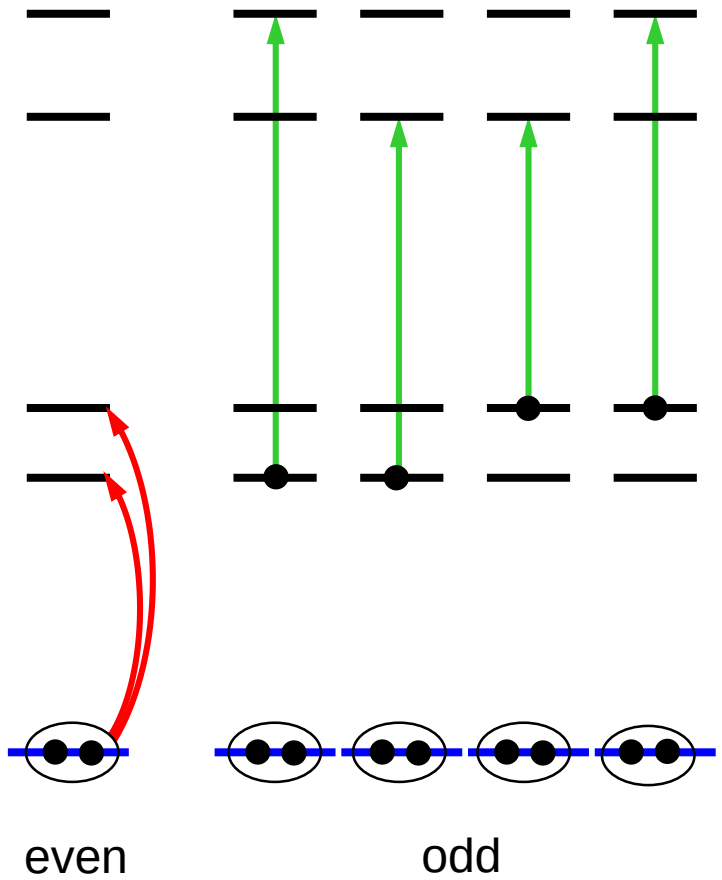
$$\tau\cos((\lambda_1-\lambda_2)\varepsilon\pm\delta)+(1-\tau)\cos((\lambda_1+\lambda_2)\varepsilon x_r)=$$
$$\cos(2\arccos\varepsilon-(\lambda_1+\lambda_2)\varepsilon)$$



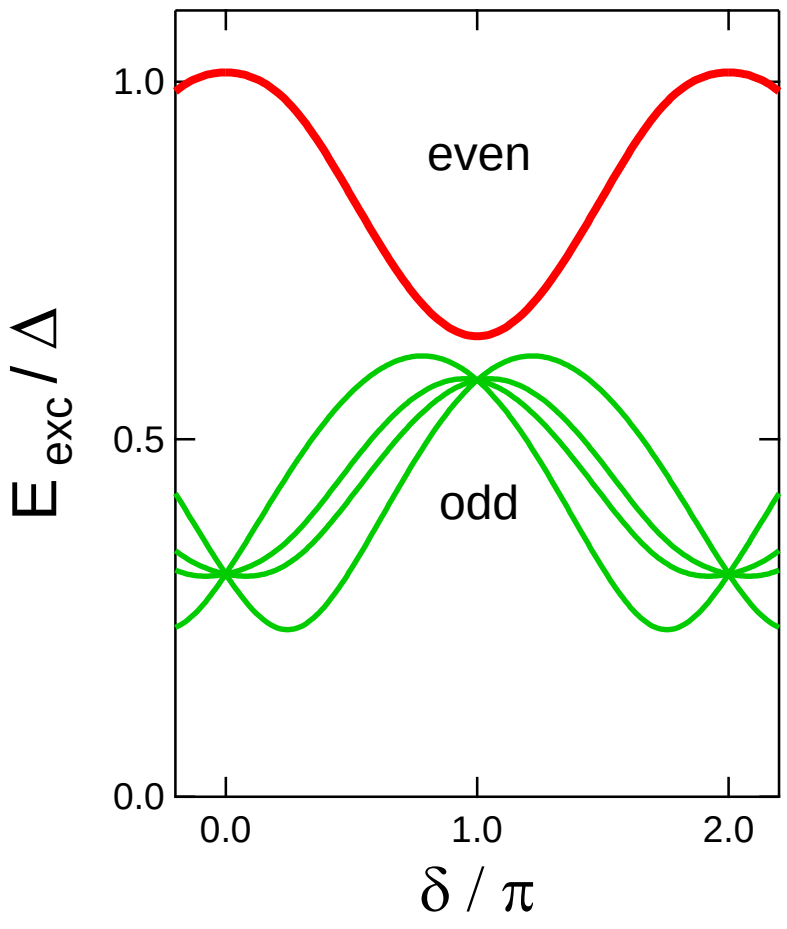
FROM ABS TO ABSORPTION SPECTRUM



Andreev bound states



Transitions at a given phase



Absorption spectrum