

Topological insulators in 1D

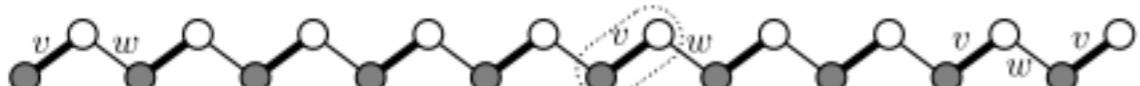
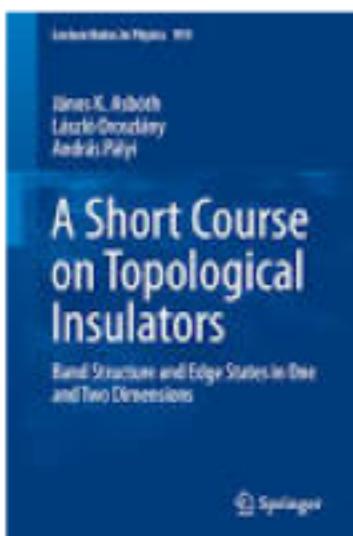
Andras Palyi

Budapest University of Technology and Economics

School on Quantum Materials, Braga, 2018.10.10.

Part 1

Introduction to topological insulators

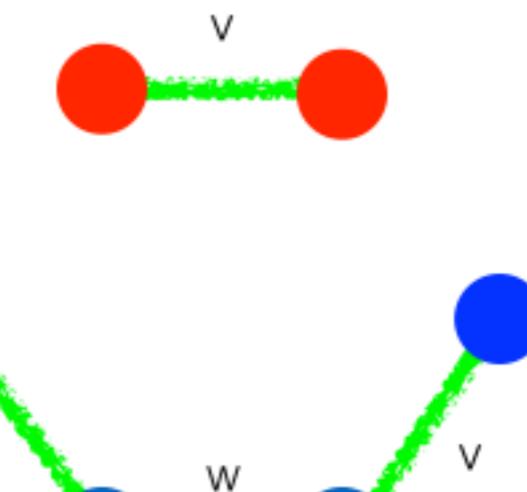


Asboth et al.
arXiv:1509.02295

(here: Chapter 1)

Part 2

Poor man's
topological quantum memory



with:

Janos Asboth, Peter Boross, Laszlo Oroszlany, Gabor Szechenyi

Part 1

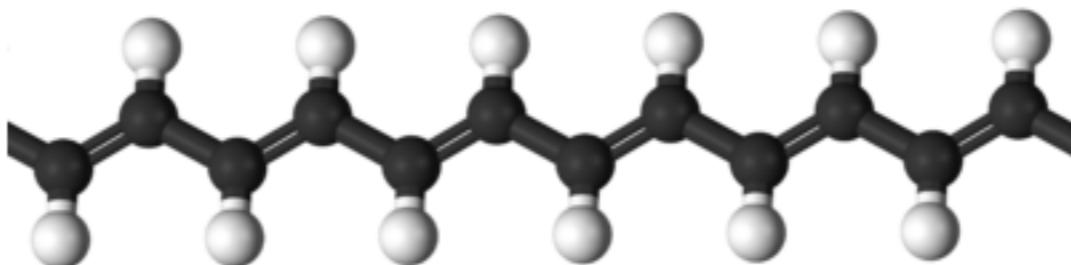
Introduction to topological insulators

**If the bulk has nontrivial topology,
then the edge has disorder-resistant bound states
(‘bulk-boundary correspondence’)**

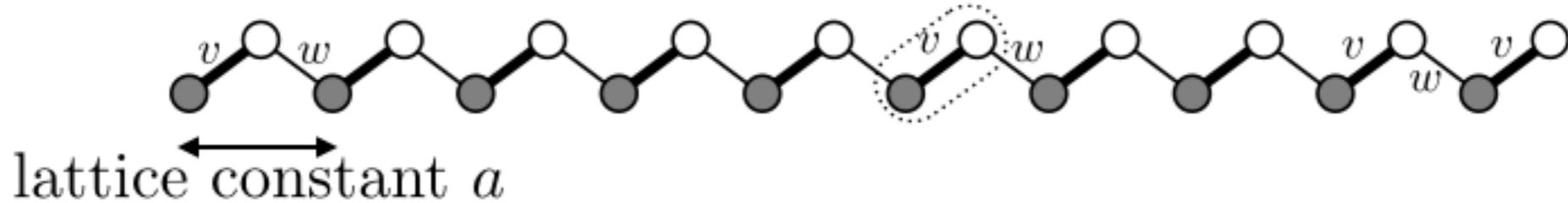
SSH is a tight-binding toy model for polyacetylene

Barisic et al., PRL 1970
Su et al., PRL 1979

Polyacetylene



Su-Schrieffer-Heeger (SSH) model of polyacetylene



Real-space tight-binding SSH Hamiltonian:

$$\hat{H} = v \sum_{m=1}^N (|m, B\rangle \langle m, A| + h.c.) + w \sum_{m=1}^{N-1} (|m+1, A\rangle \langle m, B| + h.c.).$$

intracell hopping

intercell hopping

For N=4:

$$H = \begin{pmatrix} 0 & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 & w & 0 \\ 0 & 0 & 0 & 0 & 0 & w & 0 & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \end{pmatrix}$$

k-space Hamiltonian maps unit circle to complex plane

Real-space tight-binding SSH Hamiltonian:

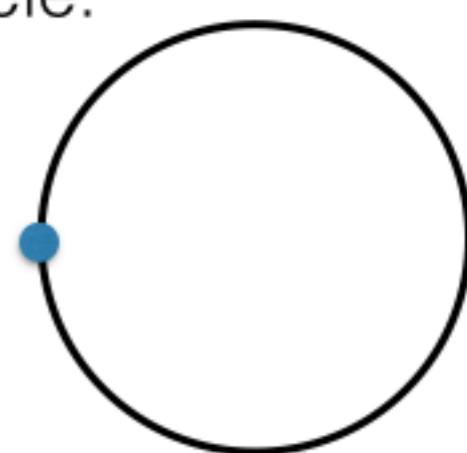
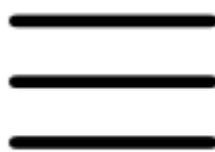
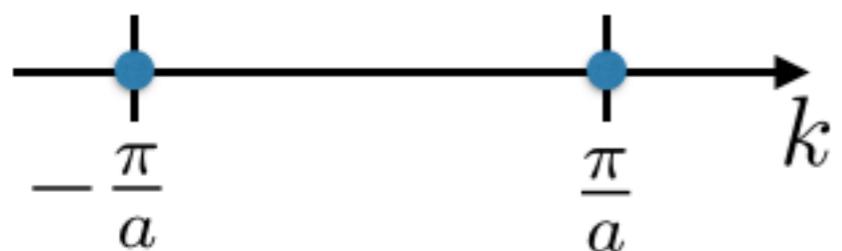
$$\hat{H} = v \sum_{m=1}^N \left(|m, B\rangle \langle m, A| + h.c. \right) + w \sum_{m=1}^{N-1} \left(|m+1, A\rangle \langle m, B| + h.c. \right).$$

k-space SSH Hamiltonian:

$$H(k) = \begin{pmatrix} 0 & v + we^{-ik} \\ v + we^{-ik} & 0 \end{pmatrix} = \mathbf{d}(k) \cdot \boldsymbol{\sigma}$$

$\mathbf{d}(k) = \begin{pmatrix} v + w \cos k \\ w \sin k \end{pmatrix}$

Brillouin zone of a 1D crystal is equivalent to the unit circle:



$$f_{v,w} : \text{unit circle} \rightarrow \mathbb{C}, k \mapsto v + we^{-ik}$$

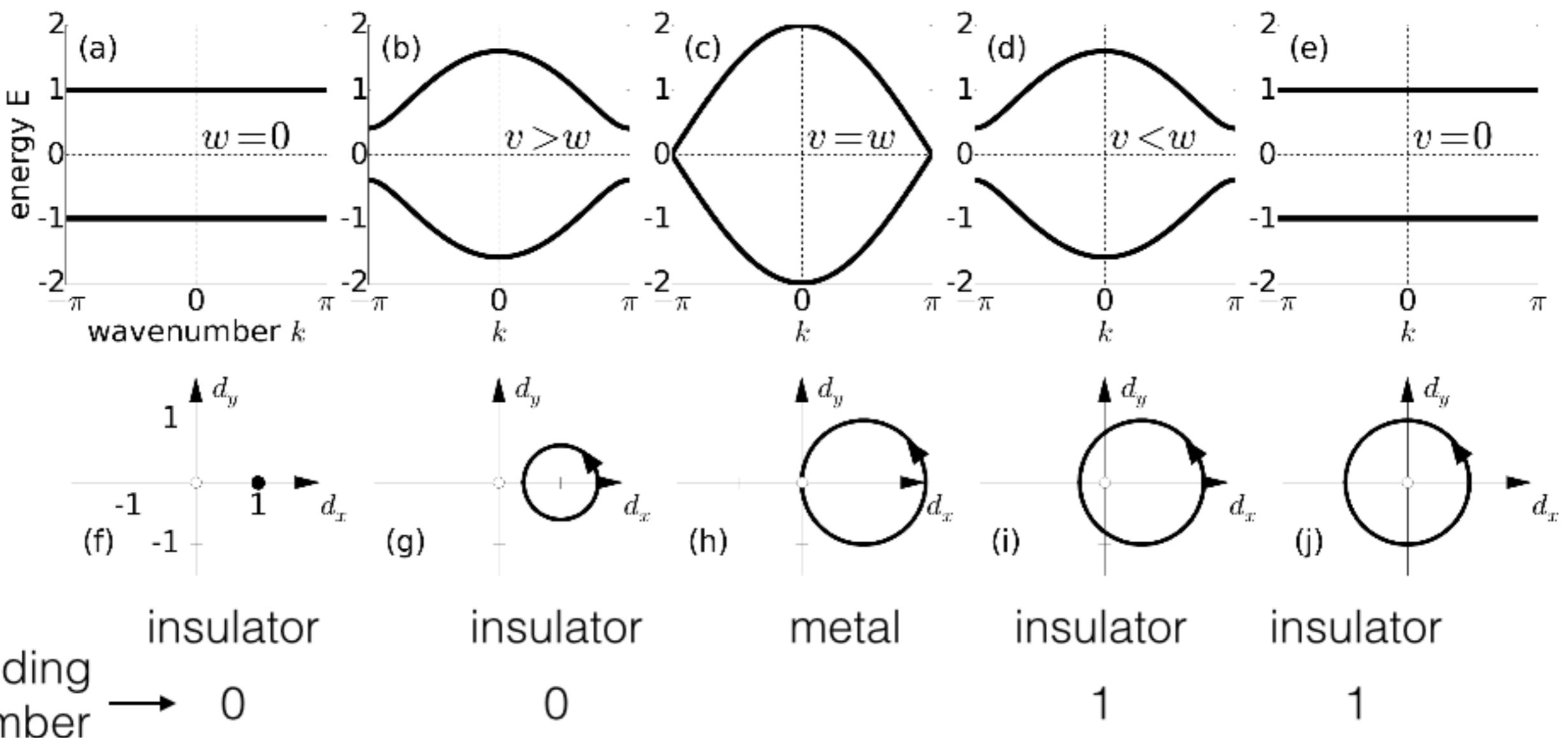
An insulating SSH Hamiltonian has a topological invariant

k-space SSH Hamiltonian:

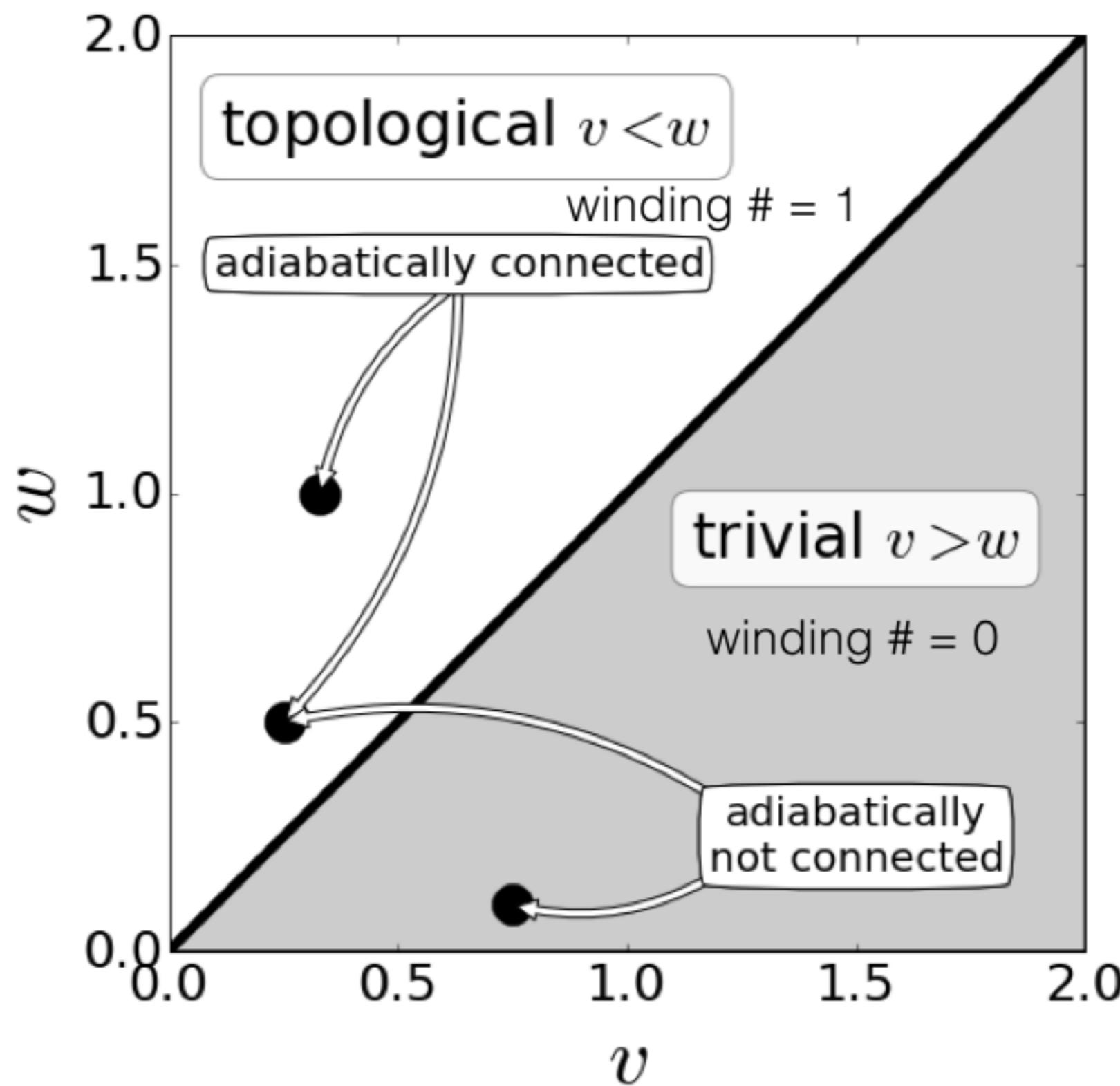
$$H(k) = \begin{pmatrix} 0 & v + we^{-ik} \\ v + we^{ik} & 0 \end{pmatrix}$$

band structure, valence (-) and conduction (+) bands:

$$E(k) = \pm |f_{v,w}(k)| = \pm |v + we^{-ik}| = \pm \sqrt{v^2 + w^2 + 2vw \cos(k)}$$



SSH parameter space has two topological phases



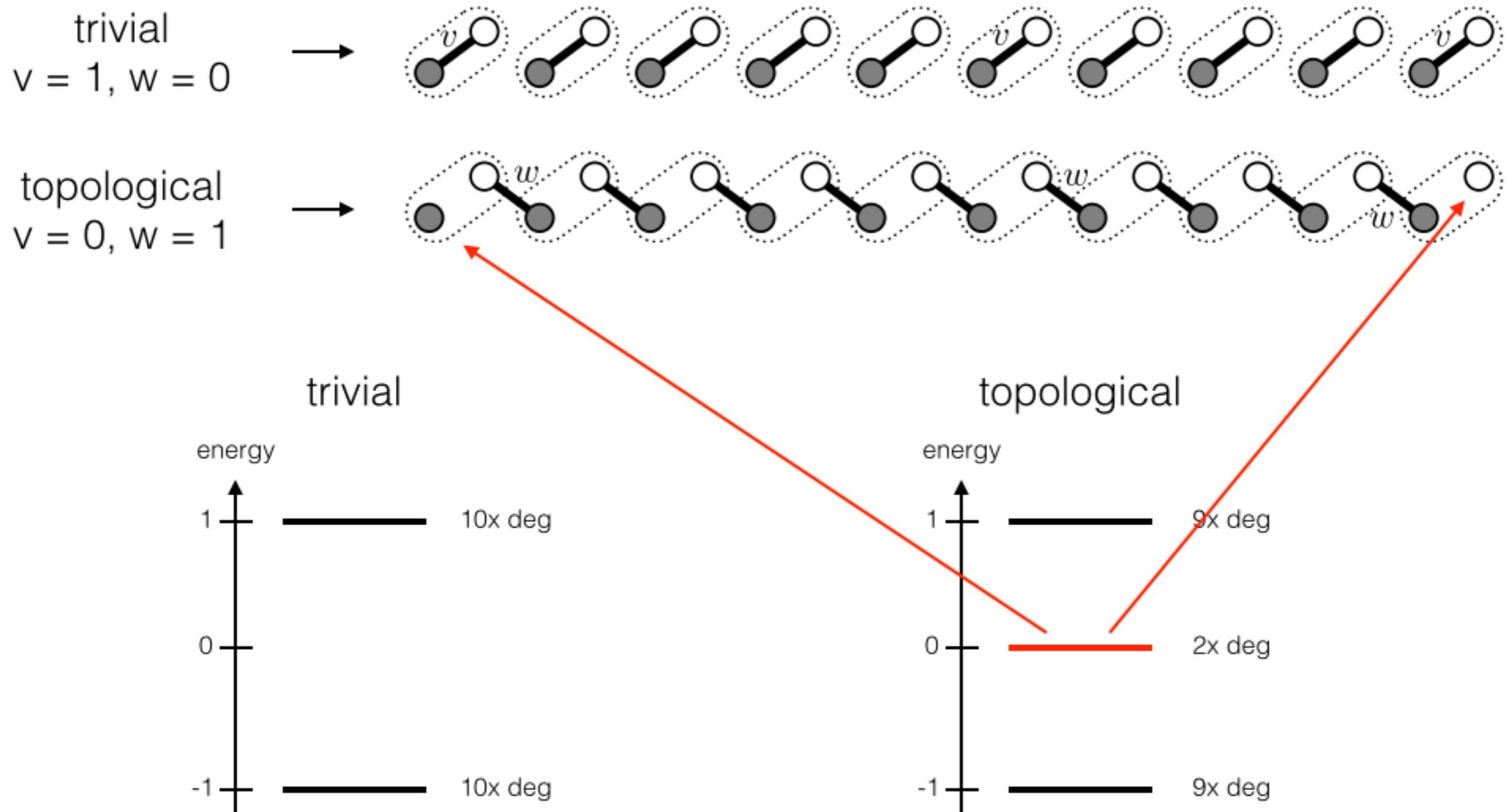
Part 1

Introduction to topological insulators

**If the bulk has nontrivial topology,
then the edge has disorder-resistant bound states
(‘bulk-boundary correspondence’)**

Zero intracell hopping implies zero-energy states at edges

Fully dimerized limits of the SSH Hamiltonian:



SSH Hamiltonians have chiral symmetry

Definition: a Γ local unitary operator is a *chiral symmetry* if $\Gamma H \Gamma^\dagger = -H$

SSH Hamiltonians have chiral symmetry:

For example, $N = 4$: $H = \begin{pmatrix} 0 & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & 0 & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 & w & 0 \\ 0 & 0 & 0 & 0 & 0 & w & 0 & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \end{pmatrix}$

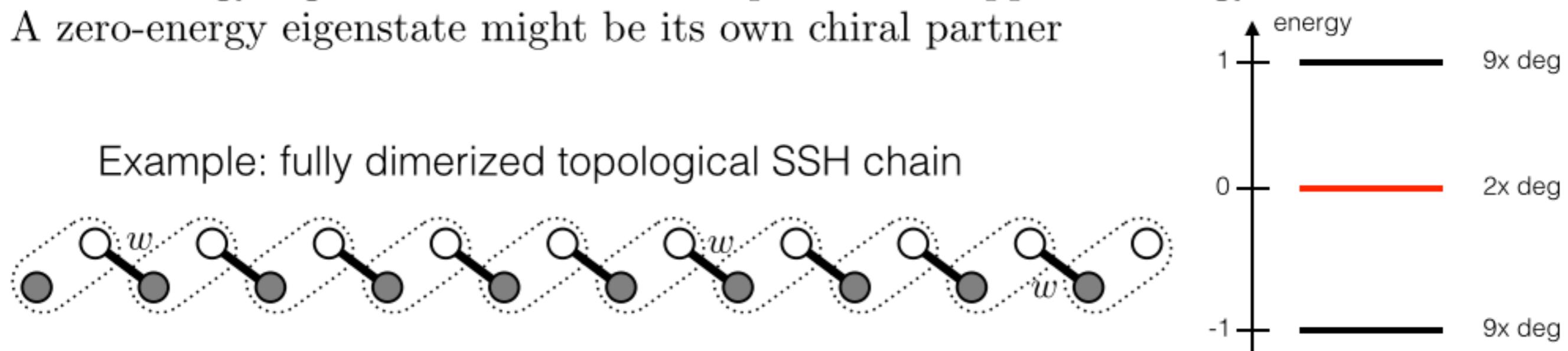
$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

Consequences of chiral symmetry

Up-down symmetric energy spectrum: $H\psi = E\psi$ implies $H(\Gamma\psi) = -E(\Gamma\psi)$

Finite-energy eigenstates have a ‘chiral partner’ at opposite energy

A zero-energy eigenstate might be its own chiral partner



Example: fully dimerized topological SSH chain

Chiral symmetry implies edge states in topological SSH

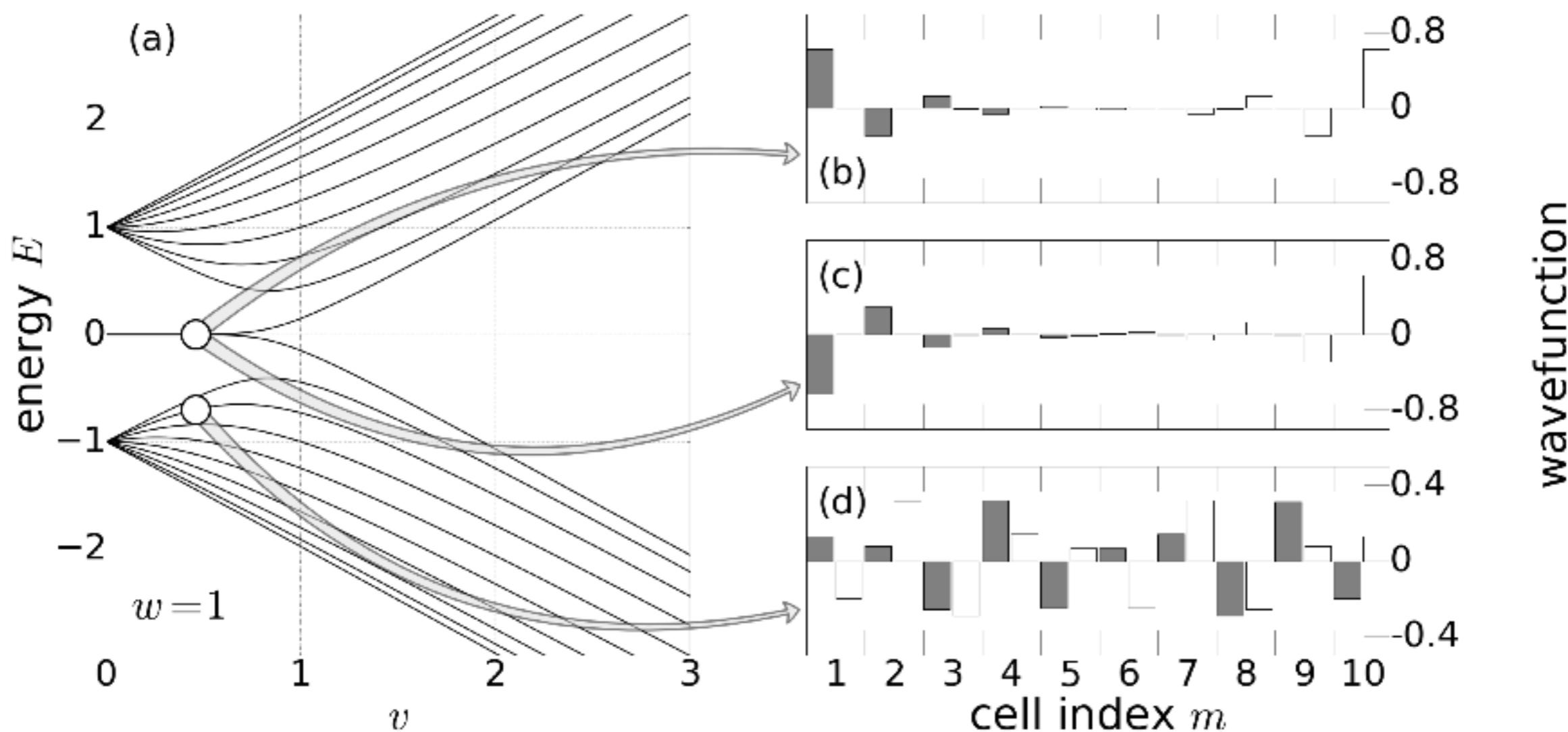
Take long fully dimerized topological SSH chain ($v=0$, $w=1$).

Switch on a uniform intercell hopping v .

Does the zero-energy edge state survive?

It does: its energy sticks to zero due to chiral symmetry.

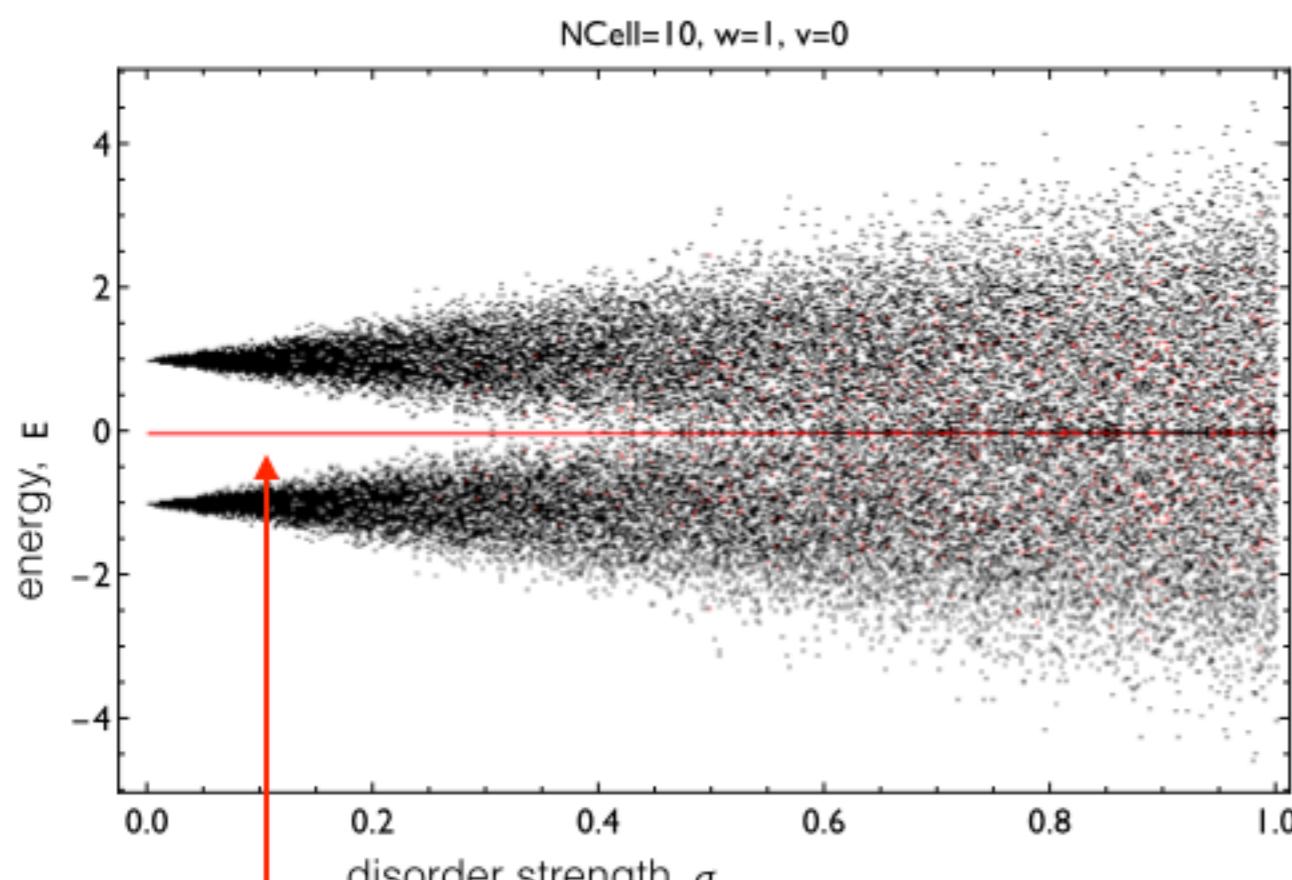
The energy can leave zero only if the left and right edge states hybridize.



bulk-boundary correspondence

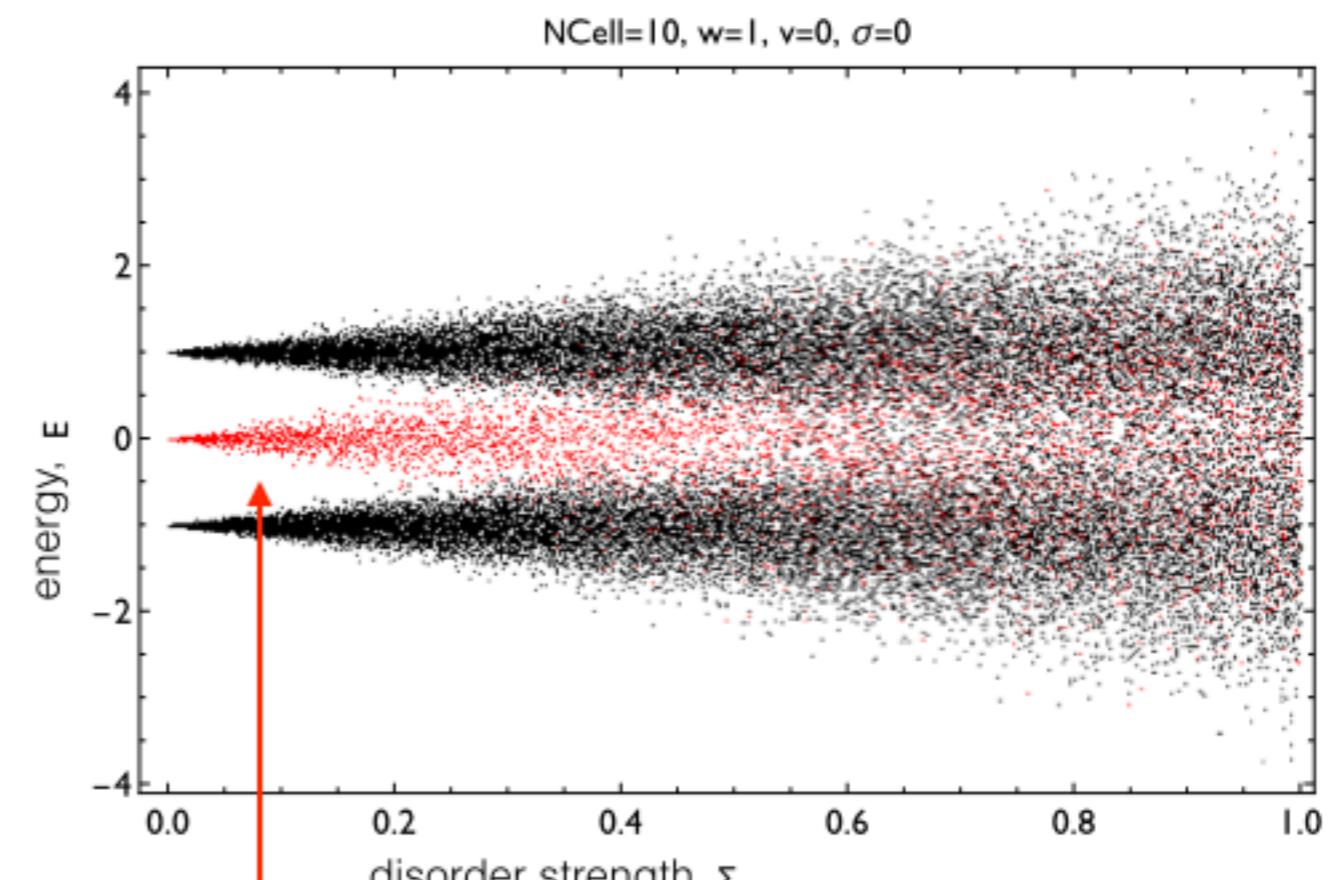
Edge states are robust against chiral-symmetric disorder

Hopping disorder
(respects chiral symmetry)



zero-energy edge states
survive disorder

On-site disorder
(breaks chiral symmetry)



zero-energy edge states:
energy gets scattered by disorder

SSH is one creature in the zoo of topological insulators

Cartan \ d	spatial dimensions			
	1	2	3	
<i>Complex case:</i>				
A	0	\mathbb{Z}	0	quantum Hall & quantum anomalous Hall effects
AIII	\mathbb{Z}	0	\mathbb{Z}	
<i>Real case:</i>				
AI	0	0	0	
BDI	\mathbb{Z}	0	0	SSH model
D	\mathbb{Z}_2	\mathbb{Z}	0	
DIII	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	
AII	0	\mathbb{Z}_2	\mathbb{Z}_2	quantum spin Hall effect
CII	$2\mathbb{Z}$	0	\mathbb{Z}_2	
C	0	$2\mathbb{Z}$	0	
CI	0	0	$2\mathbb{Z}$	

Table from A. W. W. Ludwig, Physica Scripta (2016)

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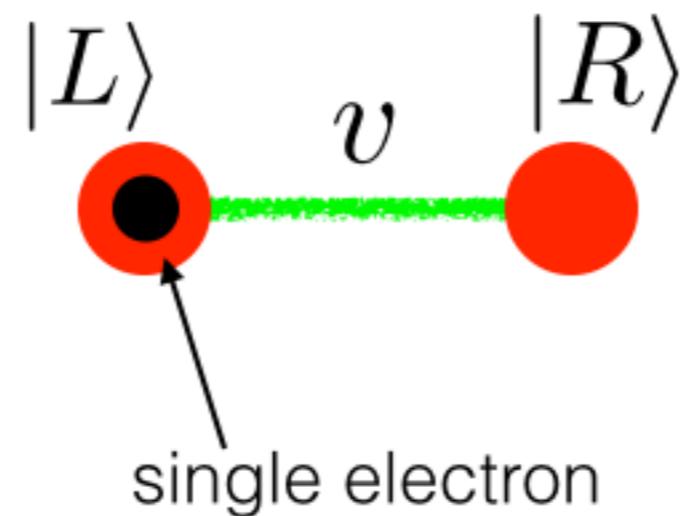
Part 2

Poor man's topological quantum memory

An almost noiseless qubit is obtained by putting together many noisy qubits

The charge qubit

two sites



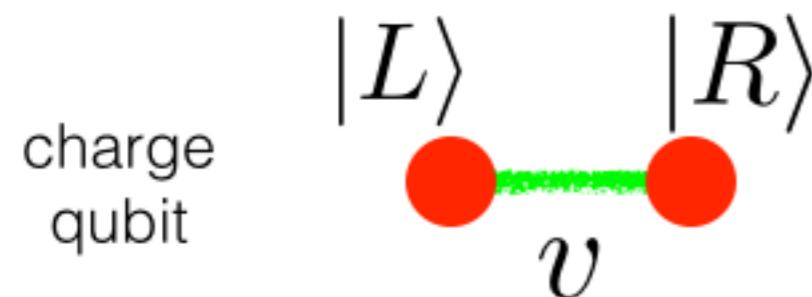
$$|\psi_0\rangle = \alpha |L\rangle + \beta |R\rangle$$

$$H = \begin{pmatrix} \epsilon & v \\ v & -\epsilon \end{pmatrix}$$

v : hopping amplitude
 ϵ : on-site energy difference

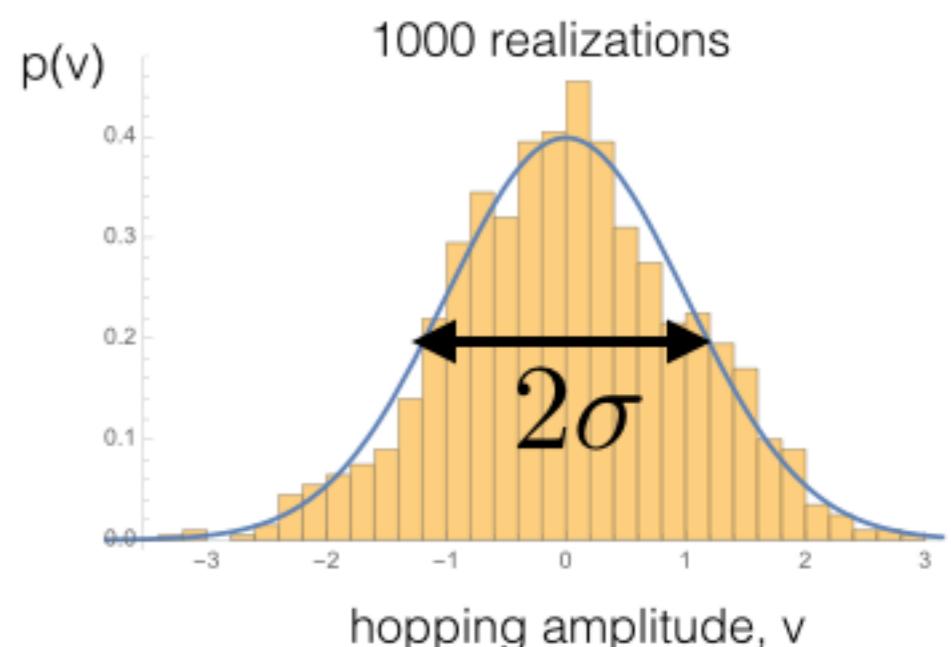
To preserve the state, set $\epsilon = v = 0$.

Hopping noise erases information in the charge qubit



charge
qubit

v : quasistatic Gaussian
random hopping amplitude

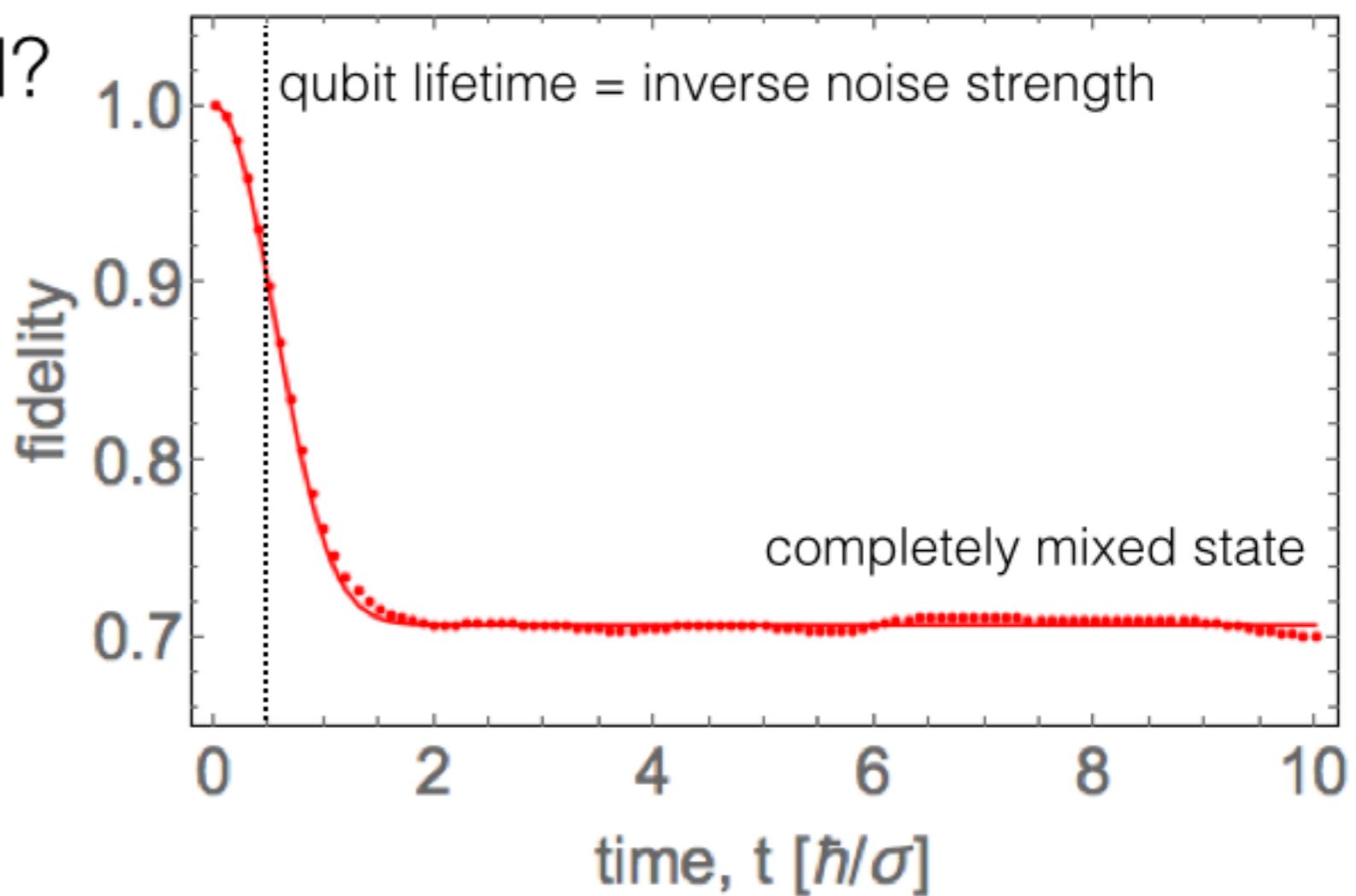


Use initial state $|L\rangle$

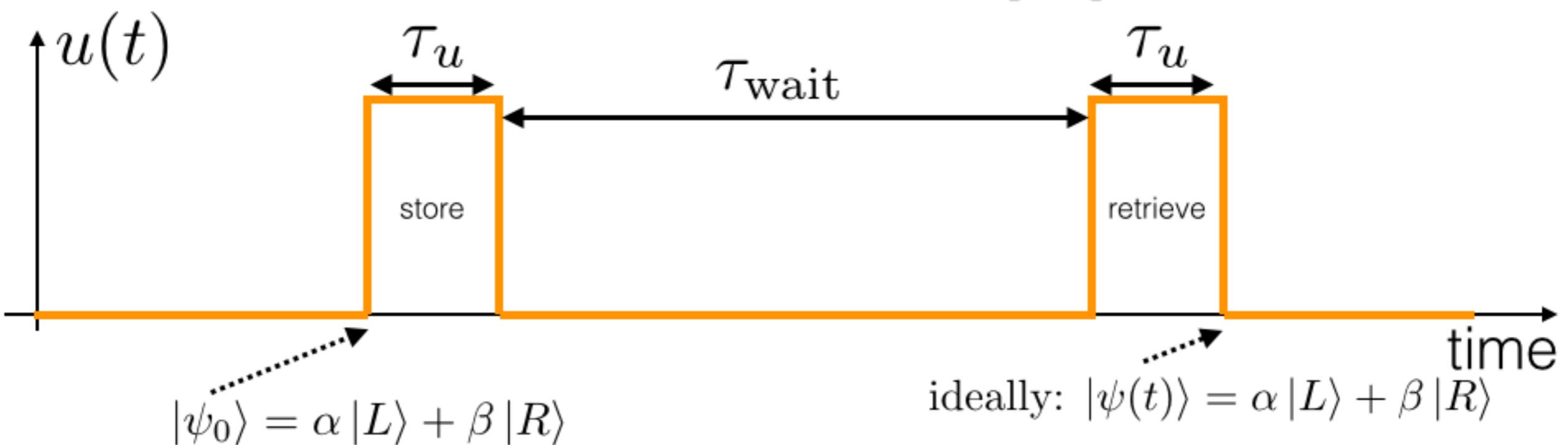
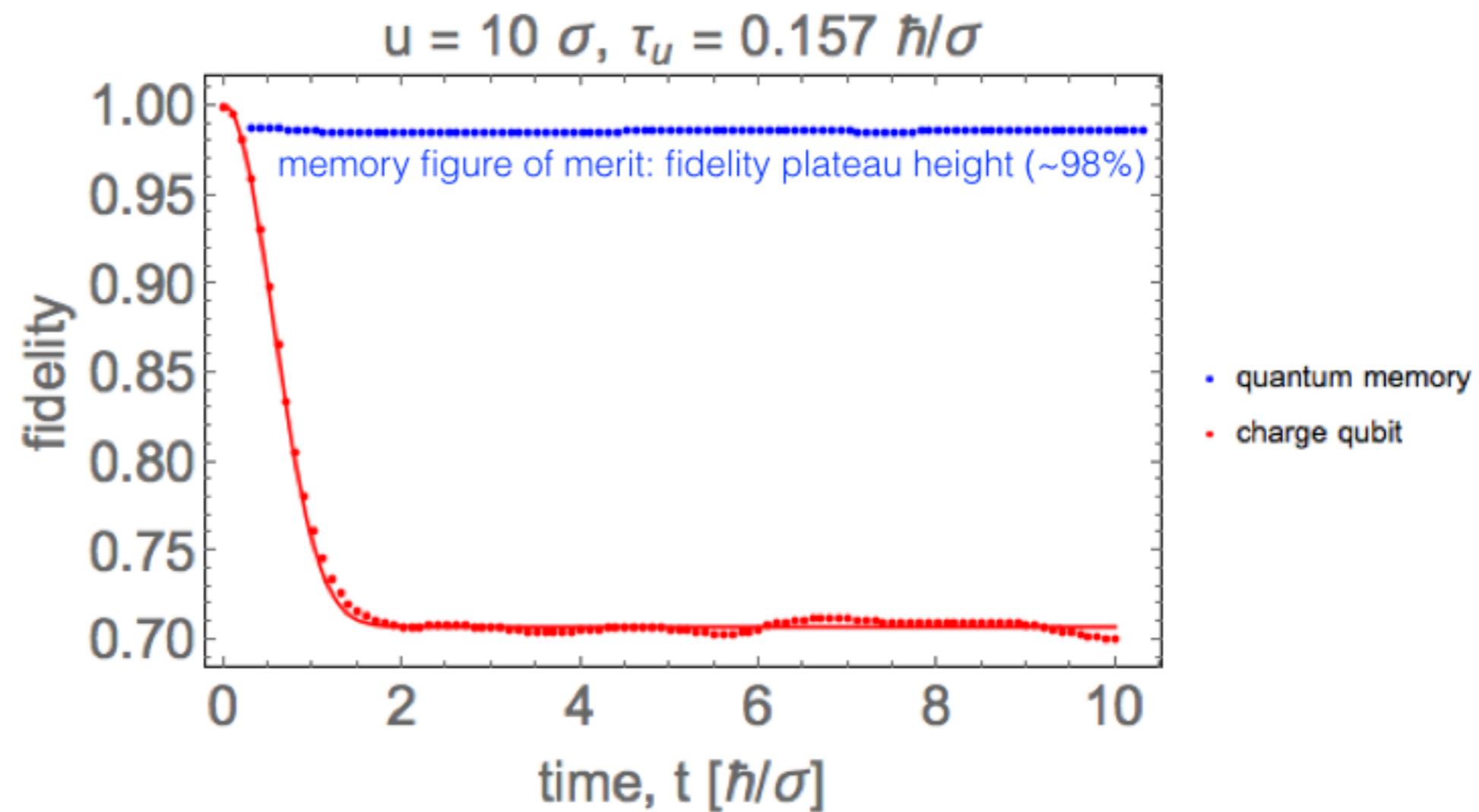
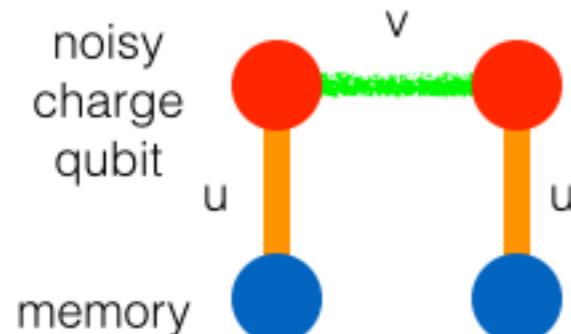
How well is it preserved?

fidelity:

$$F(t) = \sqrt{\int_{-\infty}^{\infty} dv p(v) |\langle L | \psi^{(v)}(t) \rangle|^2}$$



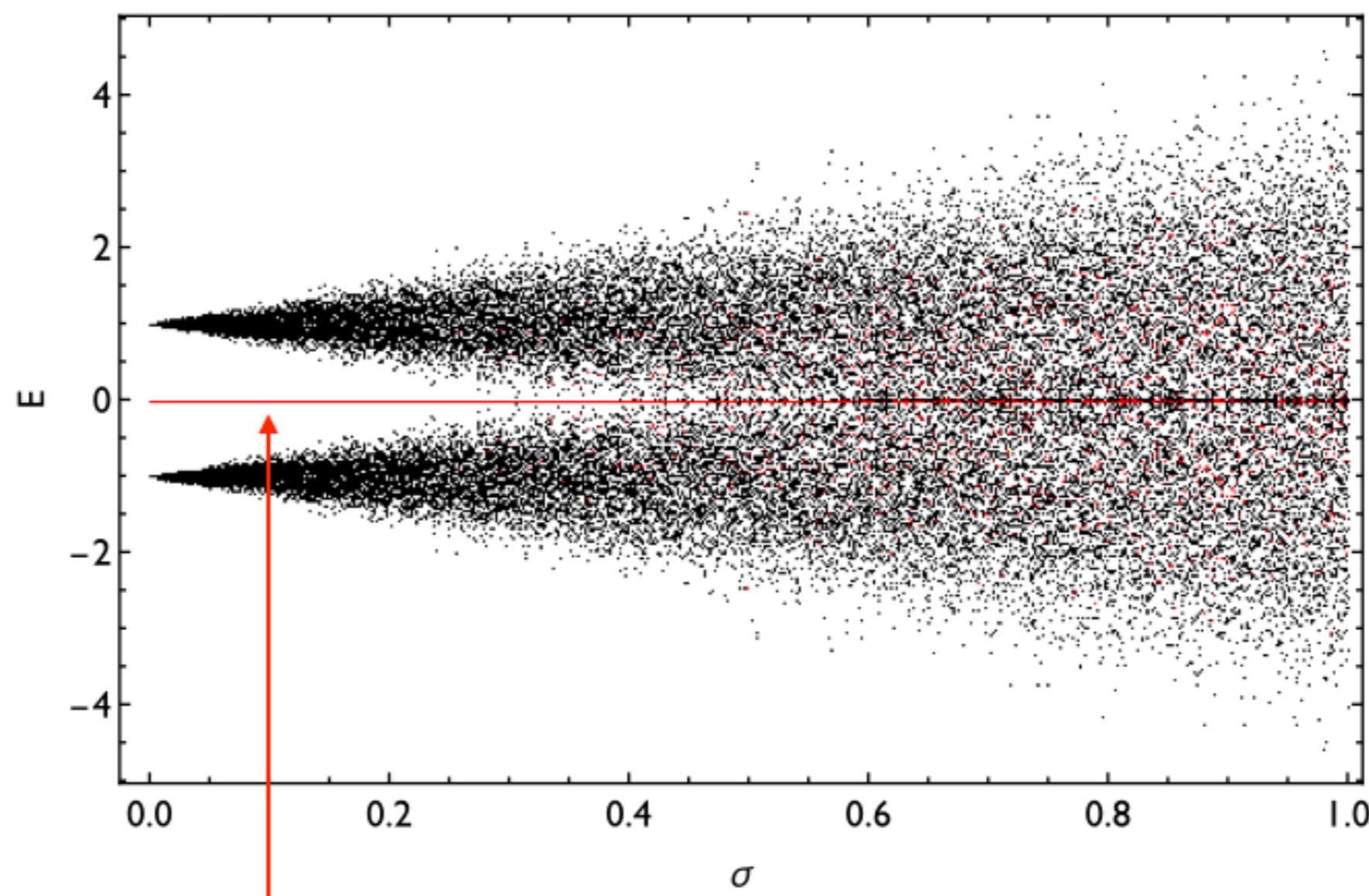
Information can survive if transferred to a less noisy qubit



Zero-energy SSH states are protected from hopping noise

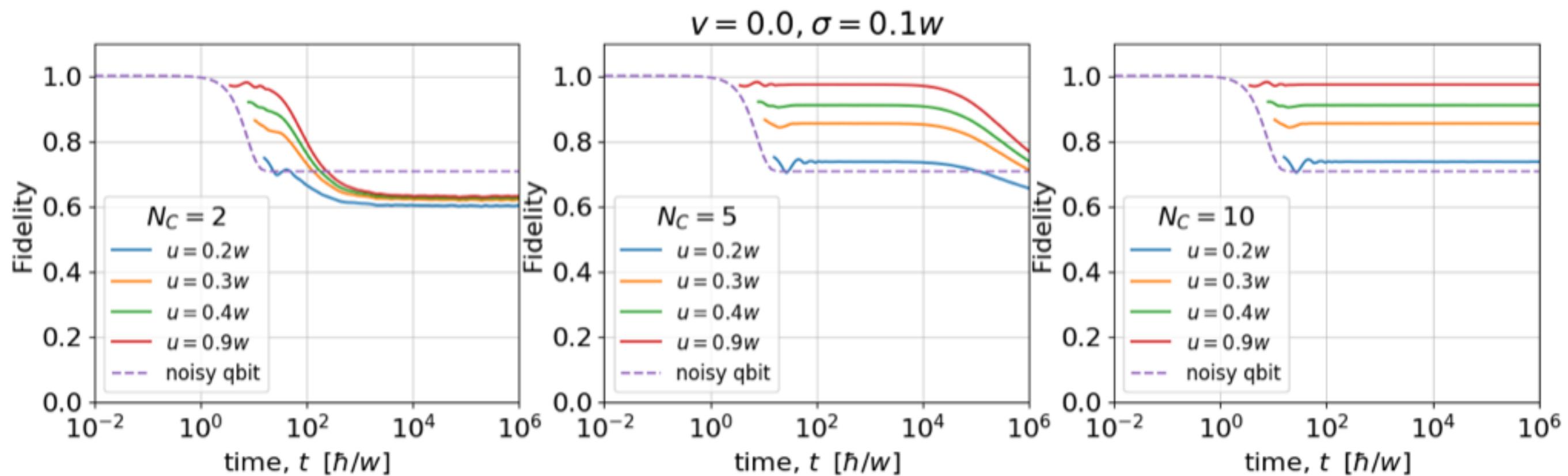
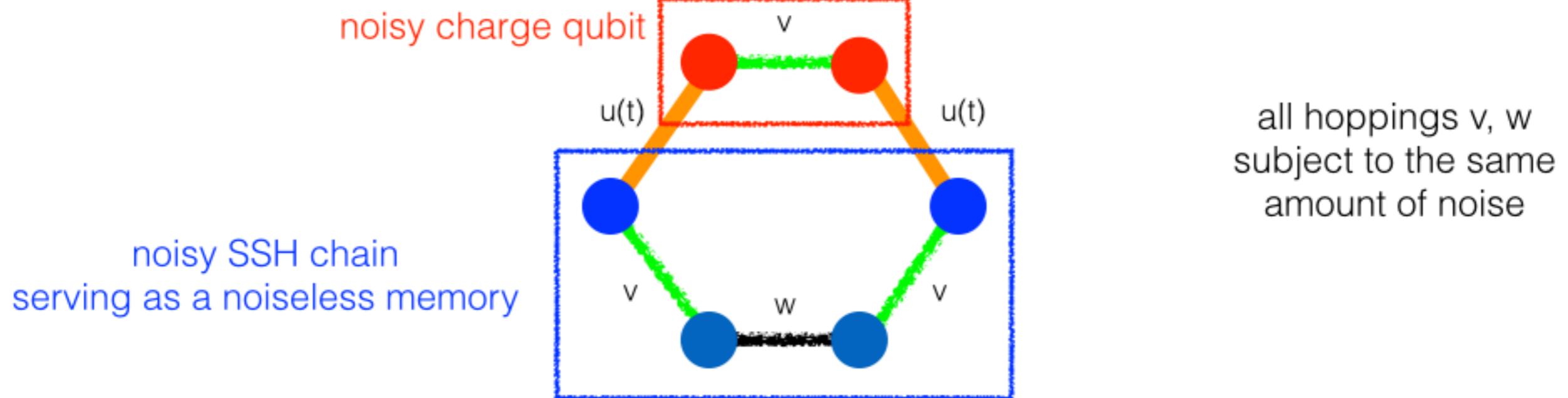


NCell=10, w=1, v=0



zero-energy edge states survive hopping disorder

SSH chain with hopping noise is a good quantum memory



memory figures of merit: height & duration of fidelity plateau

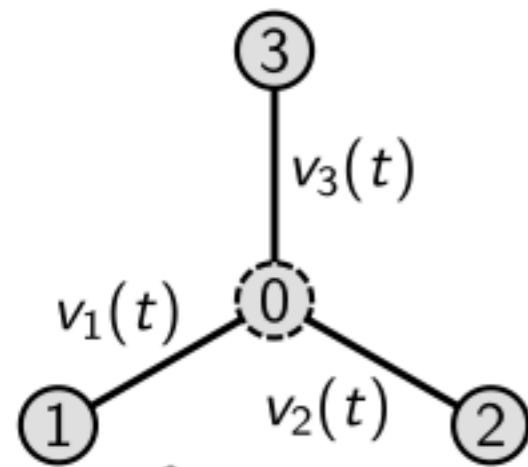
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Poor mans' topological quantum gate from the SSH model

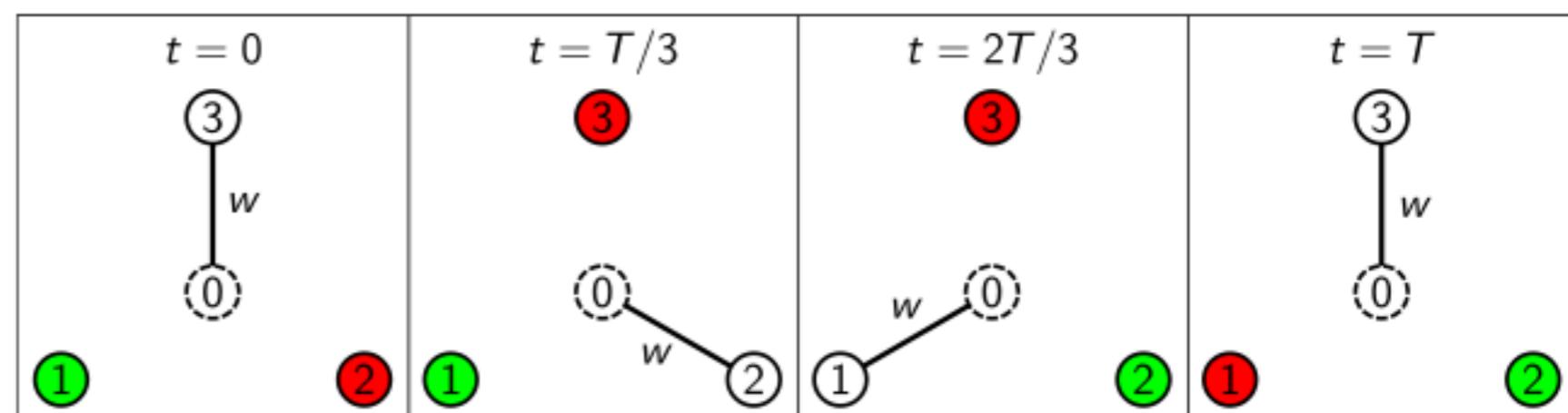
4-site minimal model:



$$\mathcal{H}(t) = \sum_{i=1}^3 [v_i(t) |0\rangle \langle i| + \text{h.c.}]$$

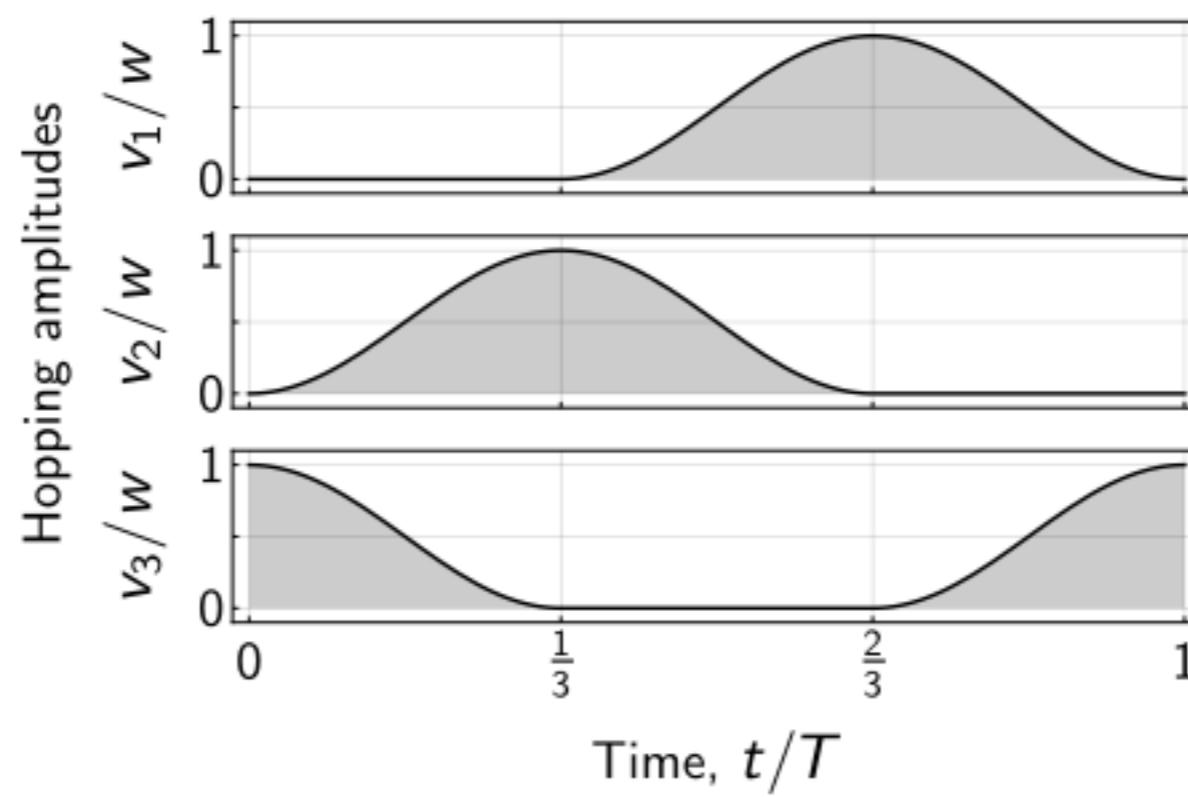
Exchange process:

$$\mathcal{H}(0) = \mathcal{H}(T)$$

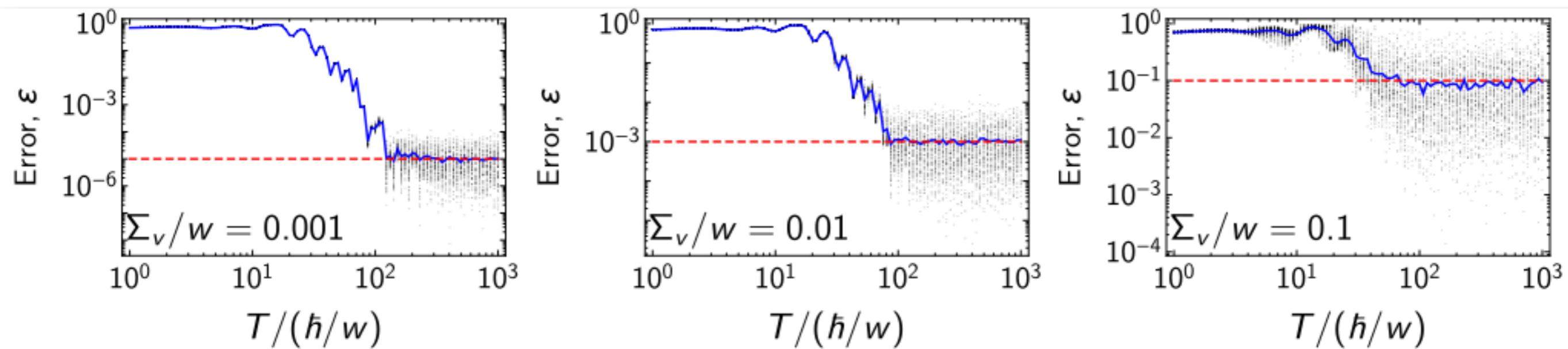


w: maximum hopping during the process

Hopping amplitudes:



Role of hopping disorder



Realization with optical waveguides?

